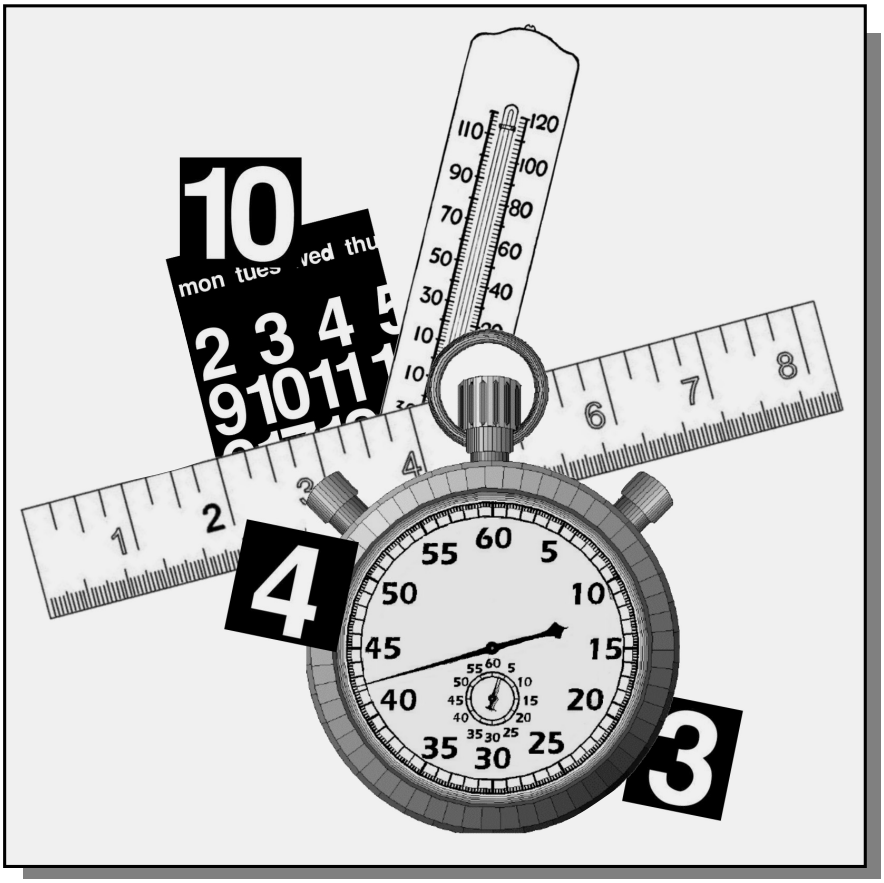
Basic Math Skills



Chapter 2

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Publisher and Managing Editor
Ronald E. Johnson, Ph.D.

Writers
Kermit Northcutt
David Browder

Technical Editor
David Browder

Content Editor / QA
Nancy Johnson

Production Manager
H. Michael Young

Page Composition
Debbie L. Dial
Kathy L. Dial
Pamela K. Hurst
H. Michael Young

Graphic Artists
Kathy L. Dial
Pamela K. Hurst
Kristin Odom
Lydia Seals
Don Ward
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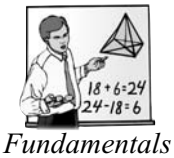
"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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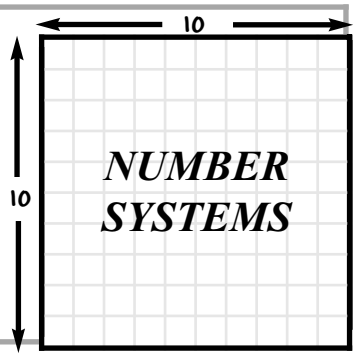
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Lesson 1

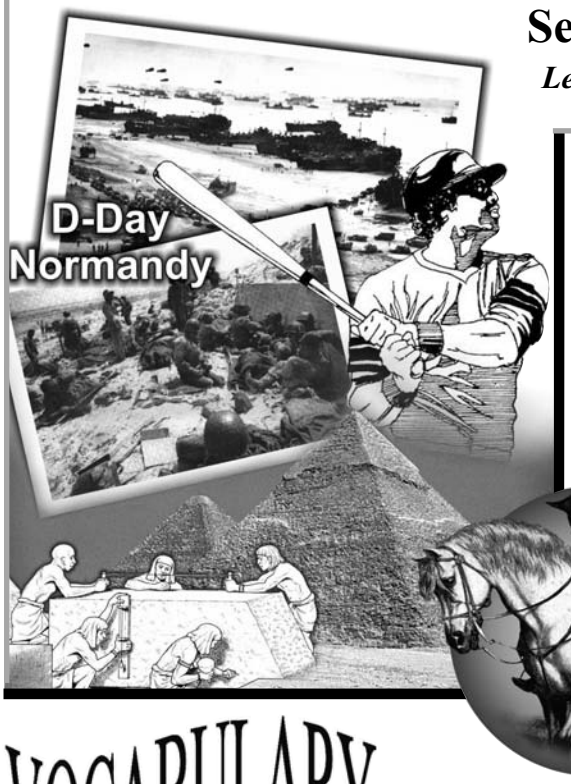
Fundamentals

Chapter 2



Section 1

Lessons 1-5





Professor K

A trip to the Middle East gave Professor K an opportunity to tell his family some of the history surrounding numbers. The symbols we use today for numbers did not always look like they do now. ***Archeologists*** had to ***decipher*** ancient writings to understand certain symbols which represented each number.

Since the time when man first inhabited Earth, he has used numbers. The first mathematics was ***tallying*** (counting) possessions. Each civilization developed its own symbols for the representation of numbers. Today, symbols used for numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are a combination of Hindu and Arabic symbols. Therefore, this system of numbers is called Hindu-Arabic.

You have seen people count on their fingers. The ancient Chinese had a complicated counting system in which each joint, knuckle, and part of the hand represented a different number, including place values for numbers.

Numbers have not always been represented in such an easy fashion as we know today. The Babylonians used triangular-shaped symbols stacked on top of each other to represent the numbers 1 through 9, and a sideways "V" to represent the number 10. At the time the pyramids were being built, the ancient Egyptians used what you may know as a variation of stick numbers. The number, 10, was an upside down "U", called a ***heel bone.***

The Mayan civilization used a series of dots for the numbers 1 - 4. A straight line represented the number five. The numbers, 6 - 9, were represented by the straight line for 5, and a dot added for each number up to 9. Two straight lines stood for the number, 10.

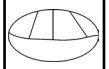
VOCABULARY

Archaeologists: scientists who study past human life and cultures by the recovery and study of material evidence, especially graves, buildings, pottery and other artifacts
Decipher: to read, interpret, or convert from code
Tallying: counting
Heel bone: a symbol used by the Egyptians to represent the number ten
Hindu-Arabic number system: combination of Hindu and Arabic number symbols used by the world today; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

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The Hebrews and Greeks used letters of their alphabets for each number. The Romans used what we know as Roman Numerals. **Table 2.1** shows what these symbols looked like in each civilization.

Table 2.1

Hindu-Arabic	0	1	2	3	4	5	6	7	8	9	10
Babylonian											
Egyptian											
Greek		α	β	γ	δ	ε	ϖ	ζ	φ	θ	ι
Roman Numeral		I	II	III	IV	V	VI	VII	VIII	IX	X
Mayan		•	••	•••	••••	____	• ____	•• ____	••• ____	•••• ____	____ ____

Notice in **Table 2.1** that only the Mayan mathematics had a symbol for zero. Both the ***Hindu and Arabic number systems*** had a symbol for zero that looked almost exactly like the symbol the combined Hindu-Arabic system uses today.



Counting numbers, natural numbers: the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9…
Set: a collection of objects; in mathematics a set is placed within brackets { }
Whole numbers: the counting numbers plus zero; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9…
Number line: a graphic representation of numbers
Greater than: to indicate something is larger than another object; in mathematics the symbol, > , is used to signify greater than; 6 > 4
Less than: to indicate something is smaller than another; in mathematics the symbol, < , is used to signify less than; 3 < 7

In modern mathematics, the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and so forth, are called ***counting numbers***, or ***natural numbers***. When zero is added to the ***set*** of counting numbers, the new set of numbers is called ***whole numbers*** {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 …}.



DEFINITION: "..."

The "..." (3 dots) following a series of numbers such as the whole numbers, {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10...} mean that the numbers continue on indefinitely in the same pattern as the previous numbers.

Example Set 1 shows how each pattern shown will continue to the next term.

Example Set 1

{2, 4, 6, 8, 10, 12, 14…} the next number in the series would be 16, since the numbers are increasing by two each time.
{6, 12, 18, 24, 30…} the next number in this series would be 36, since the numbers increase by 6.

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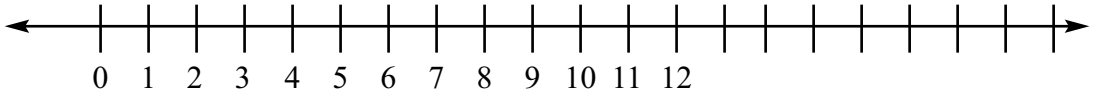
Ancient systems of tallying used symbols such as dots, triangles, or sticks. Obviously the more of these symbols used, the larger the number.

Consider two brothers who may have lived in ancient Arabia. If Ahab has 5 camels and his brother, Rauel, has 3 camels, then Ahab has the greater number of camels.



Ahab | | | | | (Each tally mark represents one camel.)
Rauel | | |

When looking at Hindu-Arabic numbers you can see which numbers are greater than, or larger than another number. The best way to see this mathematically, however, is with the use of a ***number line***, as shown below.



The number line above is a graphic representation of the whole numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Notice the arrow on the end of the number line signifies that the numbers could continue on beyond 12.

You already know that 5 is ***greater than*** 2 (5 > 2). When you look at a number line, you see that 5 is to the right of 2. The number 2 is also said to be ***less than*** 5 (2 < 5), because 2 is to the left of 5.



DEFINITION: GREATER THAN

One number is said to be greater than a second number if the first number is to the right of the second number on a number line. The mathematical symbol for greater than is >.

6 > 1, 9 > 4, 35 > 33, 256 > 200



DEFINITION: LESS THAN

One number is said to be less than a second number if the first number is to the left of the second number on a number line. The mathematical symbol for less than is <.

5 < 7, 2 < 9, 22 < 56, 145 < 157

Apply what you have learned about ancient methods of counting and the number line by working the activities for this lesson.

LIFE PRINCIPLE



"For you see, each day you are loved more. Today more than yesterday and less than tomorrow."

—Rosemonde Gerard

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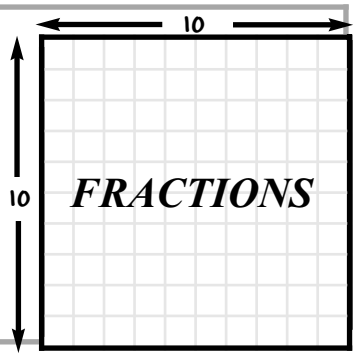
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Lesson 2

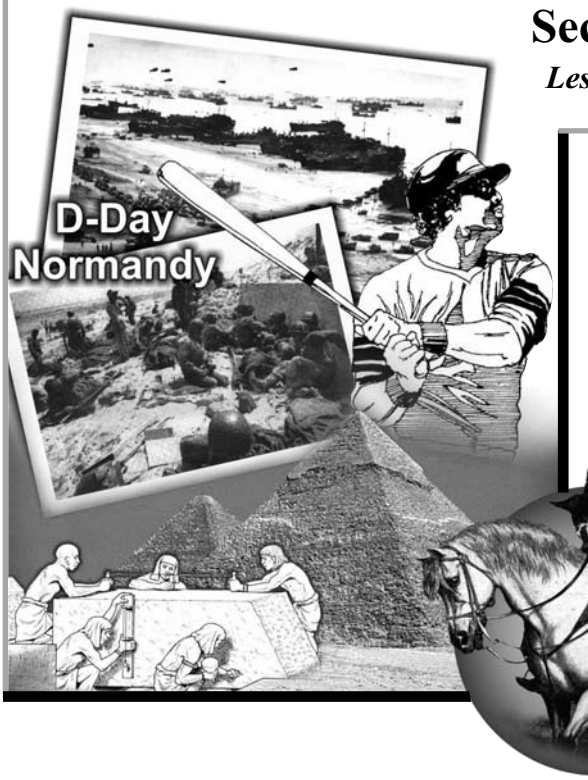
Fractions

Chapter 2



Section 1

Lessons 1-5





Professor K

Remember how Professor K thinks life is a word problem. Another aspect of life for Professor K is that life is a series of opportunities to teach math and mathematical principles. When at a pizza parlor in his hometown, Professor K was reminded of another important form of numbers that you use each day: ***fractions***. You do not always deal with complete or whole measurements of items. When eating a pizza, you don't eat the whole pizza all at once; the pizza is cut into small pieces or parts (usually eight or ten). Milk is sold in gallons (the whole) and half-gallons (the part). The fruit juice you drink is often sold in half-gallon containers. The odometer on a car divides miles into parts using decimals for the fractional part of a mile that you have traveled.

History provides examples of the use of fractions. The story of Cabeza de Vaca is laced with parts of wholes or fractions. The captain of de Vaca's expedition split the group of 600 men into two equal groups of 300 men each. This means he divided the entire group (the whole) in half (the part).

A battle in history that gives us another look at fractions was the invasion of France by Allied forces during World War II (WWII). The Allied forces suffered 40,000 casualties of the 600,000 men who charged the beaches of Normandy. This translates into a fraction in which one-fifteenth of the whole invasion force became casualties.
In this chapter you will learn how fractions and decimals are related to one another. You will learn to change fractions to decimals and vice-versa. You will also learn how to perform

VOCABULARY

Fraction: a numerical representation of a part of a whole
Numerator: the number on top of the fraction bar
Fraction bar: the line that divides the top and bottom of a fraction
Denominator: the number under the fraction bar
Proper fraction: the numerator is smaller than the denominator
Improper fraction: a fraction in which the numerator is larger than the denominator

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the four basic mathematical operations on fractions, as well as how decimals and fractions are used with money, and the two types of systems of measurement, English and Metric.

Remember, Professor K knows that you, too, can do math when you take one small step at a time and learn each principle as you go.

FRACTIONS

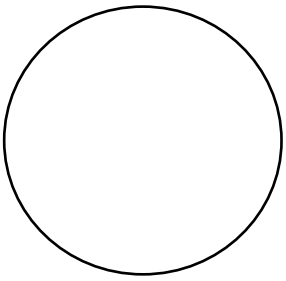
The first civilizations to use fractions were located in the region of the Babylonians and Egyptians. These civilizations primarily used what are called unit fractions, or fractions with a one in the ***numerator***. You will learn of these fractions as well as fractions that contain numbers other than one in the numerator. Study the following example carefully. Notice where the different parts of a fraction are in relationship to each other.

Example Set 1

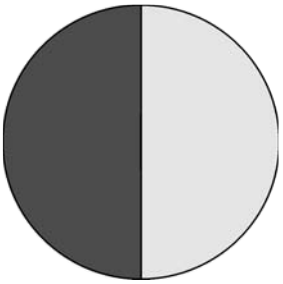
$$\frac{1}{2} \quad \begin{matrix} \text{numerator} \\ \textit{fraction bar} \\ \textit{denominator} \end{matrix} \quad 1/2$$

Example Set 1 is a numeric representation of one part of something which was cut into two pieces. The denominator shows into how many pieces the whole was cut. The numerator shows how many pieces are required. In **Example Set 1**, notice that the whole was cut into two parts (denominator), and one of those parts (numerator) was required or set apart. **Example Set 2** shows a graphic or picture representation of the fraction $\frac{1}{2}$.

Example Set 2



One whole pie



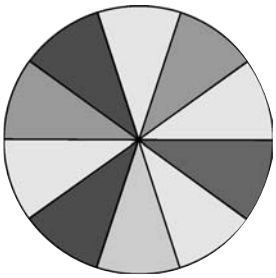
a pie cut into two pieces or two $\frac{1}{2}$s (halves) of a pie

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A pizza cut into 10 slices would look like the picture in **Example Set 3 a)**. **Example Set 3 b)** shows what the pizza would look like with 3 slices left if you were to take away 7 of the ten slices.

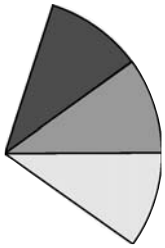
Example Set 3

a)



whole pizza cut into 10 slices

b)



three slices of pizza left from the pizza that was cut into ten slices

In **Example Set 3 a)** the fraction represented by all ten slices of pizza still in the pan is $^{10}/_{10}$. Notice that the number of pieces into which the whole was cut is 10 (denominator). The number of pieces required is 10 (numerator). Therefore, you still have 1 whole pizza.

In **Example Set 3 b)**, the fraction represented is $^{3}/_{10}$. This tells you that the whole was cut into ten pieces (denominator) and three pieces are required (numerator). The proper way to write $^{3}/_{10}$ in words is three-tenths.

RULE 1

A fraction has a value equal to 1 when the denominator and the numerator are the same number.

$\frac{2}{2}=1$ $\quad \frac{5}{5}=1$ $\quad \frac{25}{25}=1$ $\quad \frac{100}{100}=1$ or

$2\div 2=1$ $\quad 5\div 5=1$ $\quad 25\div 25=1$ $\quad 100\div 100=1$



Rule 1 is true because fractions are another method to signify division of two numbers. The denominator divides into the numerator.

RULE 2

A fraction with a zero, 0, in the numerator has a value of zero, 0.

$\frac{0}{4}=0$ $\quad \frac{0}{1}=0$ $\quad \frac{0}{9}=0$ $\quad \frac{0}{56}=0$ $\quad \frac{0}{598}=0$

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Rule 3

Division by zero has no meaning or value. Division by zero is undefined. Therefore, a fraction can never have a denominator of zero (0).

$$\frac{8}{0} = \text{not possible or undefined}$$

$$\frac{87}{0} = \text{not possible or undefined}$$

Rule 4

Any number can be written as a fraction with a 1 in the denominator.

$$2 = \frac{2}{1} \qquad 7 = \frac{7}{1} \qquad 66 = \frac{66}{1} \qquad 1257 = \frac{1257}{1}$$

Definition: Proper Fraction

A ***proper fraction*** is a fraction in which the denominator is larger than the numerator.



Example Set 4

a) $\frac{3}{4}$ a) $^3/_4$ is a proper fraction. *3* (the numerator) is less than *4* (the denominator).

b) $\frac{8}{5}$ b) $^8/_5$ is not a proper fraction because *8* (the numerator) is greater than *5*, (the denominator)

Definition: Improper Fraction

A fraction that is not a proper fraction is an ***improper fraction***. If the numerator is larger than the denominator, the fraction is considered to be improper.

Example Set 4 b) shows $^8/_5$ to be an improper fraction, since *8* is larger than *5*. The fractions, $^6/_3$, $^7/_2$, $^9/_5$, $^{81}/_7$, and $^{59}/_{31}$ are also improper fractions.

Professor K believes that proper and improper things exist in life. Each of us needs to make every attempt always to do what is right or proper.

In the next lesson you will learn how to change fractions to decimals, and then add and subtract numbers that include decimals. Demonstrate your understanding of proper and improper fractions by working the activities for this lesson.

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LIFE PRINCIPLE



"It is never right to do wrong, and never wrong to do right."

—Anonymous

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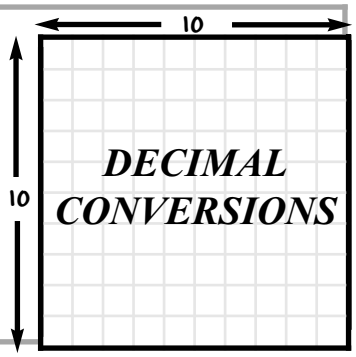
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Lesson 3

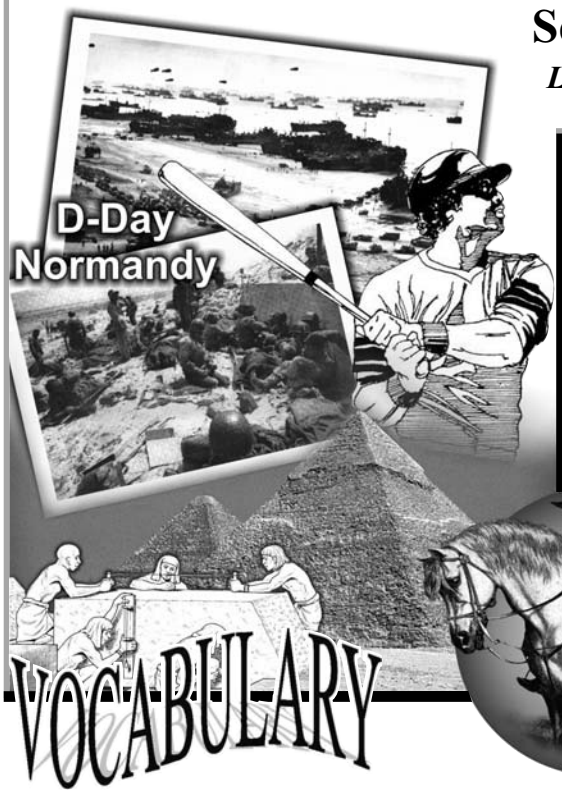
Decimals

Chapter 2



Section 1

Lessons 1-5





Professor K

In the past, baseball was considered by many as America's number one pastime. Professor K, along with most Americans, remembers stories of the baseball legends of the 1900s, "Babe" Ruth, and Ty Cobb. These two men as well as every other baseball player had a ***batting average***. Batting averages are calculated with fractions. After the batting average is calculated, the answer is shown in ***decimal form***. In the 1928 World Series while playing for the New York Yankees, "Babe" Ruth played in four games. The "Babe" was credited with a total of sixteen "at-bats" for all four games. He made ten hits during those 16 trips to the plate. As a result of his performance in the 1928 World Series, "Babe" Ruth is second on the list of hitters with 10 or more "hits-at-bats" in a World Series. In the following example, Professor K will show you how to calculate batting averages. By following the example carefully, you will learn how to change fractions into decimals without getting lost along the way.

Batting average: an average used in baseball to determine how well a batter is batting
Decimal form: a number with a decimal point (5.97)
Quotient: the answer to a division problem.
Dividend: the number that is being divided; the number under the tabletop; the number to the left of the division sign as in $6 \div 2$
Divisor: the number by which a number or thing is divided; the number outside but not on top of the tabletop; the number to the right of the division sign as in $6 \div 2$

Example Set 1

For this example, we will calculate the "Babe's" batting average in the 1928 World Series. "Babe" Ruth had 16 at-bats with 10 hits. Using the formula for determining batting average, you have the following:

$$\text{Batting Average} = \frac{\text{Number of hits}}{\text{Number of times at bat}} = \frac{10}{16}$$

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Remember from Lesson 2 of this chapter, fractions are another method to show division. The denominator divides into the numerator. In this problem *10* is divided by *16*, shown here in fraction form.

$$\frac{10}{16} = 16\overline{)10}$$

Notice that *16* cannot divide into *10*. In other words, the whole was divided into sixteen parts and only ten are required. This means we have a fractional part of the whole. Remember, a decimal point not shown is understood to be to the right of the last digit of a number. In this problem a decimal point needs to be added to the *10*. Also, in order to end the problem with no remainder and have three digits to the right of the decimal point, we add three zeros to the right of the decimal point because batting averages are figured to three decimal places. The problem now looks like this:

$$16\overline{)10.000}$$

The decimal point for the answer in a division problem is aligned the same as when adding or subtracting. In a division problem the decimal point is aligned with the decimal under the tabletop. The decimal for the ***quotient*** is placed immediately above the decimal in the ***dividend***. As the problem is worked below, notice where the decimal is placed in the quotient.

```
                    .625   <-- quotient (batting average)
(16 at bats) divisor --> 16)10.000   <-- dividend (10 hits)
                       - 96
                         40
                       - 32
                         80
                       - 80
                          0
```

"Babe" Ruth's batting average is .625. Sports announcers just say Ruth had a batting average of six hundred twenty-five, but in reality the decimal is to the left of the first digit (.6).

In **Example Set 1**, the statement was made that three places were needed to the right of the decimal. Digits to the right of a decimal point have place value just like digits to the left of the decimal point. The following table shows the value of places to the right and left of the decimal point. Notice the fraction or decimal labels to the right of the decimal point always end in “ths,” while whole numbers to the left of the decimal point end in “s.”

Table 3.1

Ten thousands	Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths	Ten thousandths
10,000	1,000	100	10	1	•	$^1/_{10}$	$^1/_{100}$	$^1/_{1,000}$	$^1/_{10,000}$
← Whole numbers					•	Fractions/Decimals →			

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Looking again at "Babe" Ruth's batting average for the 1928 World Series, .625 is read as six hundred twenty-five thousandths, since the thousandths place is the location of the last digit to the right (the *5*). Look at the following examples to become familiar with how to say or write in words the meaning of the numbers.

Example Set 2

Decimal	Fraction	Words
1,245.38	$1245\frac{38}{100}$	One thousand, two hundred forty five and thirty-eight hundredths.
35.763	$35\frac{763}{1000}$	Thirty-five and seven hundred sixty-three thousandths
56,789	$\frac{56,789}{1}$	Fifty-six thousand, seven hundred eighty-nine
.6	$\frac{6}{10}$	six tenths
5.9	$5\frac{9}{10}$	five and nine tenths

Notice in **Example Set 2** that the word, "and" is used to signify location of the decimal point.

By calculating "Babe" Ruth's batting average, you changed a fraction into its decimal equivalent. Remember, to change any fraction into its decimal equivalent, you divide the numerator by the denominator. Sometimes you will have a decimal and want to change it to fraction form. Look again at **Example Set 2**. Notice the number 35.763. The decimal portion of the number, .763, was changed to $^{763}/_{1000}$ by looking at the place value of the last digit. The digits 763 were placed in the numerator. By looking at **Table 3.1**, we see that the place value of the last digit, 3, was determined to be thousandths. The 1000 was placed in the denominator, making the fraction $^{763}/_{1000}$.

From **Example Set 2**, .6 became $^{6}/_{10}$ in the same manner. The 6 was placed in the numerator and the place value was determined to be tenths. Therefore, the ten was then placed in the denominator, producing the fraction $^{6}/_{10}$.

Table 3.2 shows the decimal equivalent for six of the most commonly used fractions. Memorizing the facts in **Table 3.2** will benefit you as you work the rest of this chapter. Keep in mind that any fraction can be changed to a decimal by dividing the denominator into the numerator.



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Table 3.2

Fraction	Decimal	Fraction	Decimal
$\frac{1}{4}$	.25	$\frac{1}{8}$	.125
$\frac{1}{3}$	.333	$\frac{1}{2}$	.5
$\frac{1}{5}$	.2	$\frac{2}{3}$	.667

In the activities for this lesson you will check your knowledge of changing decimals to fractions by figuring batting averages for various batters. Later in this chapter you will learn more about fractions and various practical methods to perform calculations with fractions.

LIFE PRINCIPLE



"If you don't know where you are going, when you get there you'll be lost."

—Yogi Berra

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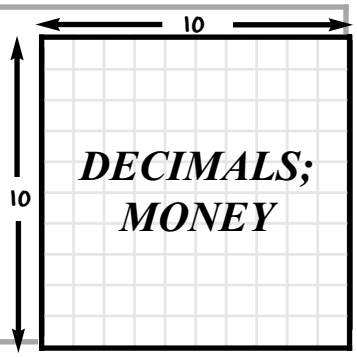
Basic Math Skills



Lesson 4

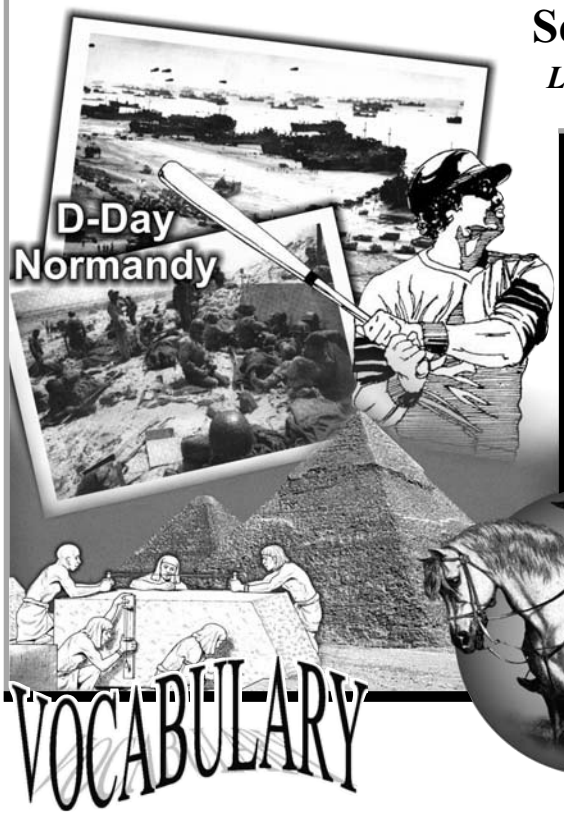
Decimals

Chapter 2

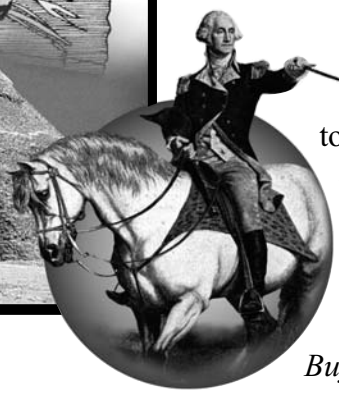


Section 1

Lessons 1-5



While considering baseball games, Professor K got hungry and also remembered a famous baseball song. Professor K will use the song and the food mentioned in it to show you how decimals are used in calculating money.



If you had attended the 1928 World Series and saw "Babe" Ruth play, you would have probably wanted to get some peanuts and Cracker Jacks™, as mentioned in the famous song about baseball, "Take Me Out to the Ballgame."

Take me out to the ballgame,
Take me out to the crowd.
Buy me some peanuts and Cracker Jacks,
I don't care if I ever get back.
'Cause it's root, root, root for the home team,
If they don't win it's a shame.
For it's one, two, three strikes you're out,
At the old ballgame.

If you wanted to know the cost to buy a ticket for the game and buy something to eat, you would have to add different amounts of ***money*** together: ***cents, nickels, dimes, quarters, dollars***. Money is written in decimal form. **Table 4.1** shows the place value of digits when talking about money.

Table 4.1

Dollar Sign	Ten thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tens of cents	Cents
$	10,000	1,000	100	10	1	•	$^1/_{10}$	$^1/_{100}$

Money: a standard used to measure something's value; official currency or coins of a government
Cents (or pennies): each cent is valued at $^1/_{100}$ of a dollar (100 cents = 1 dollar); $ .01
Nickels: each coin worth 5 cents or $^1/_{20}$ of a dollar; $ .05
Dimes: each coin worth 10 cents or $^1/_{10}$ of a dollar; $ .10
Quarters: each coin worth 25 cents or $^1/_4$ of a dollar; $ .25
Dollars: in form of "silver dollar" coins or usually paper money worth 100 cents; $1.00

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Example Set 1

For this example, we will calculate how much money you might have spent had you attended the 1928 World Series. If the cost of admission was one *dollar*, and you purchased a beverage for a *dime*, two bags of peanuts for a *nickel* each, and a box of Cracker Jacks for a dime, then what would have been the total cost of your day at the ball game?

The words, "total cost," imply an addition problem. Two bags of peanuts at a nickel each tells you to add the price of the peanuts twice. Now add the cost of all the items together. Look at the vocabulary section of this lesson to translate the words, dollar, dime and nickel, into decimals.



$1.00 cost of admission
.10 cost of a beverage
.05 cost of one bag of peanuts
.05 cost of second bag of peanuts
+ .10 cost of one box of Cracker Jacks
$1.30 total cost for your afternoon at the ball game

Remember to align the decimal points to keep the place values of the dollars, dimes, and nickels in their correct columns.

In **Example Set 1**, the total cost of your day at the ball game is written in words as one dollar and thirty cents. Notice that when writing amounts of money, only two decimal places are used to the right of the decimal. Dollars are divided into one hundred parts called *pennies* or *cents*.

Look at **Example Set 2 (a and b)**. Notice that the same rules using decimals for addition apply to the process of carrying. When necessary, the number carried crosses over the decimal, just as if the decimal were not there, into the place value column to the left.

Example Set 2a

Step 1

$$\begin{array}{r} 4.50 \\ +\ 2.76 \\ \hline \end{array}$$

In this problem *4.5* is added to *2.76*. A zero is placed in the space to the right of the *5* to mark off the *hundredths* place. (See Step 1.) Now add the single digits in each place value column, carrying whenever necessary. Place the decimal point in the sum in alignment with the other decimal points. (See Step 2.)

Step 2

$$\begin{array}{r} {\scriptstyle 1} \\ 4.50 \\ +\ 2.76 \\ \hline .26 \end{array}$$

Looking at the *hundredths* column, the *6* is added to the *0* and the sum of *0 + 6 = 6* is placed in the answer. Be sure to keep the digits in the proper column. Next, the digits *5* and *7* are added in the *tenths* column. *5 + 7 = 12*. The *2* is placed in the *tenths* place of the answer and the *1* (one group of ten tenths) is carried across the decimal into the *ones* column. (See Step 2.) (Remember, $^{10}/_{10} = 1$; as we learned in Chapter 2 Lesson 2 Rule 1)

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Step 3

```
  1
  4.50
+ 2.76
  7.26
```

The digits *1*, *4*, and *2* are added in the *ones* column. *1 + 4 + 2 = 7*. The *7* is placed in the *ones* place of the answer, giving the final sum of *7.26*. (See Step 3.)

Example Set 2b

Step 1

```
  15.456
   3.567
+  .056
```

This problem in addition has three numbers involving decimals added together. The procedure is the same. First, add the three digits in the *thousandths* column. (See Step 1.)

Step 2

```
   11
  15.456
   3.567
+  .056
  79
```

6 + 7 + 6 = 19. The digit, *9*, is placed in the *thousandths* place of the sum, and the *1* (one group of ten thousandths or one one-hundredth) is carried to the *hundredths* column to be added to the digits already there. *1 + 5 + 6 + 5 = 17*. The *7* is placed in the *hundredths* place of the sum and the *1* is carried to the *tenths* column. (See Step 2.)

Step 3

```
   111
  15.456
   3.567
+  .056
  19.079
```

In the *tenths* column, *1 + 4 + 5 + 0 = 10*. The *0* is placed in the *tenths* place of the sum, and the *1* (one group of ten tenths or one) is carried to the *ones* column, where it is added to *5* and *3*. *1 + 5 + 3 = 9*. You have nothing to carry so the *1* in the *tens* column is brought down into the *tens* place of the sum. The final sum, therefore, is *19.079*. (See Step 3.)

Subtraction of decimals also follows the same rules as normal subtraction of numbers. Remember that all the decimals, including the decimal in the difference must be kept aligned. Borrowing can occur across the decimal, just as carrying was done across the decimal. **Example Set 3** will show you an example of borrowing when decimals are involved.

Example Set 3

Step 1

```
  562.68
− 89.74
    4
```

This problem has *89.74* subtracted from *562.68*. Beginning with the *hundredths* column, *8 - 4 = 4*. (See Step 1.)

Step 2

```
    1 16
  562.68
− 89.74
   .94
```

In the *tenths* column, *7* is subtracted from *6*. Since this cannot be done, one group of ten tenths (a "one") must be borrowed across the decimal point from the *2* in the *ones* place of *562.68*. The *2* now becomes a *1*, and the *6* becomes *16*. This means that *7* is now subtracted from *16*. *16 - 7 = 9*. The *9* is placed in the *tenths* place of the answer as shown. The decimal point is placed to the left of the *9* in alignment with those decimal points above it. (See Step 2.)

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Step 3

$$\begin{array}{r} {\scriptstyle 5\ 11} \\ {\scriptstyle 1\ 16} \\ 5\not{6}\not{2}.\not{6}8 \\ -\ 89.74 \\ \hline 2.94 \end{array}$$

In the *ones* column *9* is to be subtracted from *1*. This cannot be done so one group of ten is borrowed from *6* in the *tens* place of *562.68*, changing the *6* to a *5* and the *1* to *11*. Then, *11 - 9 = 2*. The *2* is placed in the *ones* place of the answer. (See Step 3.)

Step 4

$$\begin{array}{r} {\scriptstyle 4\ 15} \\ {\scriptstyle 5\ 11} \\ {\scriptstyle 1\ 16} \\ 5\not{6}\not{2}.\not{6}8 \\ -\ 89.74 \\ \hline 472.94 \end{array}$$

Looking at the *tens* column, *8* is to be subtracted from *5*. This cannot be done so *1* group of hundreds is borrowed from the *5* in the *hundreds* place of *562.68* (making the *5* a *4*) and placed with the *5* in the *tens* place to make *15*. Then, *15 - 8 = 7*. The *7* is placed in the *tens* place of the difference. Zero is subtracted from the *4* in the *hundreds* place. *4 - 0 = 4*. The *4* is placed in the *hundreds* place of the answer. The difference of *562.68* minus *89.74* is *472.94*. (See Step 4.)

Professor K encourages you to complete the activity questions to demonstrate your knowledge of how to use decimals when calculating the cost of something.

Life Principle



"The borrower is servant to the lender."

—King Solomon

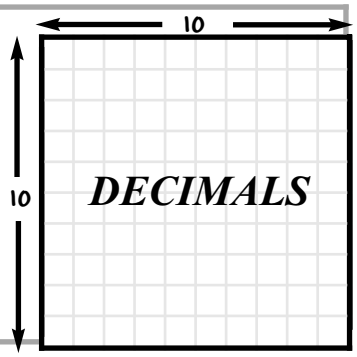
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Lesson 5

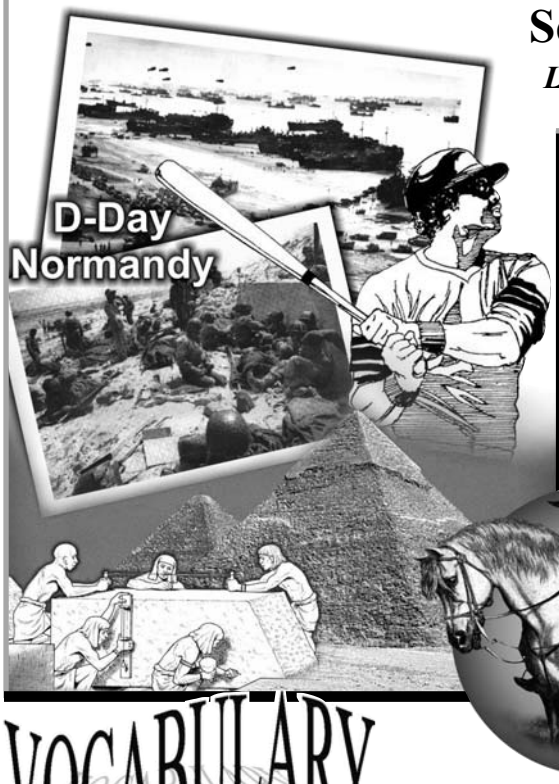
Decimals

Chapter 2



Section 1

Lessons 1-5



A story often told about ***George Washington*** is one that Professor K has referred to many times while talking with his children about truthfulness. As a young boy, George Washington began the habit of telling the truth. In the story, George Washington's father asked young George who had cut down a small cherry tree. George, the guilty person, could have easily denied the action; but telling the truth was part of George Washington's character, so he admitted that he was at fault. This attribute and others, when multiplied together, made George Washington a strong and popular leader.

In this lesson you will see that multiplication of numbers that include decimal points is carried out in much the same way as numbers that do not include decimal points. The difference in multiplying numbers with decimal points is counting the total number of decimal places to the right of all the decimal points in the factors, and putting the decimal point in the proper location in the product. Look at **Example Set 1** to see how this is accomplished.

VOCABULARY

George Washington: courageous leader in America's War for Independence and first president of the United States

Dividend: the number that is being divided; the number under the tabletop; the number to the left of the division sign as in $6 \div 2$

Quotient: the answer to a division problem.

Divisor: the number by which a number or thing is divided; the number outside but not on top of the tabletop; the number to the right of the division sign as in $6 \div 2$

Example Set 1

Step 1

$$\begin{array}{r} 2.5 \\ \times\ 3.2 \\ \hline \end{array}$$

This problem requires multiplying *2.5* by *3.2*.

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Step 2

$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ \times\ 3.2 \\ \hline 5\,0 \end{array}$$

Look first at the *ones* column, and work only with the *2* in *3.2* and the *5* in *2.5*, $2 \times 5 = 10$. The *0* is placed in the *ones* place of the first answer row, and the *1* is carried to the *tens* column to be added to the product of the digits in the *tens* column. The next step is $2 \times 2 = 4$; and $4 + 1 = 5$. The *5* of this answer is placed in the *tens* place of the first answer row. (See Step 2.) In multiplication, the decimal point is placed differently than in addition, subtraction or division (where the decimal point is immediately aligned). In multiplication, the decimal point is placed last in the process. (See Steps 5 and 6.)

Step 3

$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ \times\ 3.2 \\ \hline 5\,0 \\ 5\,0 \end{array}$$

Starting in the second answer row, mark off the *ones* place with a *0* so the next product lines up under the *3*, because *3* is in the *tens* place. Now, $3 \times 5 = 15$; the *5* is placed in the *tens* place of the second answer 6row, and the *1* is carried above the *2*, in *2.5*. (See Step 3.)

Step 4

$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ \times\ 3.2 \\ \hline 5\,0 \\ 7\,5\,0 \end{array}$$

In the next step, $3 \times 2 = 6$, and $6 + 1 = 7$. The *7* is placed beside the *5* of the second answer row. (See Step 4.)

Step 5

$$\begin{array}{r} {\scriptstyle 1} \\ 2.5 \\ \times\ 3.2 \\ \hline 5\,0 \\ +\,7\,5\,0 \\ \hline 8\,0\,0 \end{array}$$

In the next step, add the two products together, $50 + 750$ to get the final product, *800*. (See Step 5.)

Now look at the factors *2.5* and *3.2*. One digit is to the right of the decimal point in each of the factors. Add the count of these two digits together, $1 + 1 = 2$. Two total decimal places are to the right of the decimal point in both factors combined.

Step 6

800

8.00

In the product, start at the right of the last digit (*0*) and count toward the left, two places. This would place the decimal point in the final product between the *8* and the first zero as shown. (See Step 6.)

The number of decimal places in the product can be determined simply by counting the total number of decimal places to the right of the decimal point in all the factors. **Example Set 2** shows examples of this process.



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Example Set 2

Factors	Number of decimal places to the right of the decimal point in the product
3×4.56	2
56.3957×9.3	5
3×3.4×6.67	3
123.067×2.3	4

Factors	Number of decimal places to the right of the decimal point in the product
12.4×45.678	4
3×5×6	0
34×25×78×.5	1
1.398475×7	6

Notice **Example Set 2**. To find the number of decimal places to the right of the decimal point in the final product, count the total number of decimal places to the right of the decimal points in all the factors.

DIVISION

You will see that division of numbers that include decimal points is carried out in much the same way as numbers that do not include decimal points. The decimal in the answer is placed with respect to the location of the decimal in the ***dividend***. Look at the examples below to see how this is accomplished.

Example Set 3

$8.4 \div 2$

Step 1 $2\overline{)8.4}$

In this problem, *8.4* is divided by *2*. (See Step 1.) The decimal point in the ***quotient*** is placed in alignment with the other decimal point in the dividend.

Step 2

$$\begin{array}{r} 4.2 \\ 2\overline{)8.4} \\ -\,8 \\ \hline 0\,4 \\ -\,4 \\ \hline 0 \end{array}$$

divisor
dividend
quotient

In Step 2, $8 \div 2 = 4$. The *4* is placed on top of the tabletop above the *8*. Notice that $2 \times 4 = 8$ and the *8* is placed under the *8* of the dividend. When you subtract, the *4* of the dividend is brought down. (See Step 2.) Next, $4 \div 2 = 2$. Place the *2* on top of the tabletop above the *4* of the dividend. Notice $2 \times 2 = 4$. The *4* is placed under the *4* and subtracted. (See Step 2) This leaves a remainder of *0*, and the quotient is *4.2*.

Another type of problem you will encounter when dividing numbers with decimal points is one number containing a decimal point divided by another number containing a decimal point. In dividing numbers with a decimal point in both ***divisor*** and dividend, count the total number of decimal places to the right of the decimal point in the divisor. Then move

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the decimal point to the right of the last number in the divisor, making it a whole number. Move the decimal point in the dividend to the right the same number of places as you did in the divisor. Now the division can be completed. **Example Set 4** shows the process for division of such numbers.



PRINCIPLE

If the decimal point is moved to the right a given number of places in the divisor, the decimal point in the dividend must be moved the same number of places to the right.



Example Set 4

Step 1 $2.34\overline{)2.8782}$

This problem has *2.8782* divided by *2.34*. In this type of problem both the divisor and the dividend have a decimal point. (See Step 1.)

Step 2 $2.34\overline{)2.8782}$

To work this problem, the divisor cannot have any decimal places to the right of the decimal point. This requires you to move the decimal point in the divisor two places to the right. You must then move the decimal point in the dividend the same number of places to the right, in this case, two. (See Principle above.) This now makes the new division problem, *287.82 ÷ 234*. (See Step 2.)

Step 3 $\begin{array}{r} 00 \\ 234.\overline{)287.82} \end{array}$

In Step 3, *234* cannot divide into *2*; so a *0* is placed on top of the tabletop above the *2* of the dividend to mark off the place. Likewise, *234* cannot divide *28*; so another *0* is placed on top of the tabletop above the *8* of the dividend to mark off the place. (See Step 3.)

Step 4 $\begin{array}{r} 001. \\ 234.\overline{)287.82} \end{array}$

Notice that *234* divides into *287* one time (or *287* contains one group of *234*). The *1* is placed on top of the tabletop above the *7* of the dividend. The decimal point is placed to the right of the *1* on top of the tabletop above the decimal point of the dividend. (See Step 4.)

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Step 5

```
      001.23
234.)287.82
   – 234
     538
   – 468
     702
   – 702
       0
```

In Step 5, *1 × 234 = 234*. This product is placed under the dividend in the proper location and subtracted as shown. The *8* is brought down beside the *53* to make *538*. The next division problem is *538* divided by *234*. Notice that *234* divides into *538* two times (or *538* contains two groups of *234*). Place a *2* on top of the tabletop above the *8* of the dividend. *234 × 2 = 468*. Place the product, *468*, beneath *538* and subtract. The *2* is brought down from the dividend and placed beside the *70* to make *702*. Next, *234* divides *702* three times (or *702* contains three groups of *234*). Place the *3* on top of the tabletop above the *2* of the dividend. *234 × 3 = 702*. Place the product, *702*, under the difference, *702*, and subtract. The remainder is *0*. Thus, the quotient of *2.8782 ÷ 2.34* is *1.23*.

Apply what you have learned about decimals by working the activities for this lesson.

Life Principle



"Good habits multiply to build good character."

—Regi Schexnider

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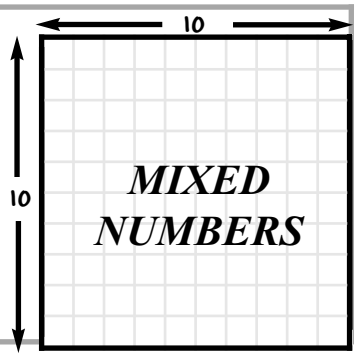
Fractions

Lesson 6

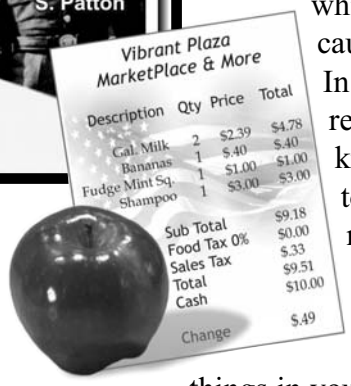
Chapter 2

Section 2

Lessons 6-10









Professor K

Many people believe you can mix a good life with bad practices and ideas. Professor K knows that is not possible. Grocers do not mix good fruit and vegetables with rotten ones. A tree designed to produce sweet fruit does not produce sour fruit. Mixing chemicals properly can produce beneficial products, while mixing chemicals improperly can cause great harm and even death. In the kitchen, a mother can mix up a really wonderful dessert. Good cooks know just what ingredients to mix together in the correct proportions to make the final product taste good. Learning the proper procedure for mixing chemicals, combining baking ingredients and arranging things in your life is important to the outcome. After learning the proper procedure, you must practice what you have learned so that by ***habit*** you do the right thing at the right time.

Professor K believes that the same is true in your study of mathematics. Studying mathematics can help you in other areas of life. Mathematics is a structured science that can help you to do other tasks in an orderly and methodical fashion. Practicing proper methods and procedures is very important in the study of mathematics. In this lesson Professor K will teach you more about fractions.

VOCABULARY

Habit: something you do naturally without having to think about it; second nature
Mixed number: a number consisting of a whole number and a fraction combined; ($1\frac{1}{2}$ or $2\frac{1}{4}$).
Equivalent: having the same value; equal to

FRACTIONS - MIXED NUMBERS

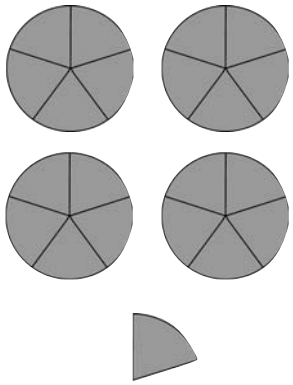
Sometimes fractions are combined (mixed) with whole numbers. These types of numbers are called ***mixed numbers***. You may have $2\frac{1}{2}$ gallons of milk in your refrigerator, or $10\frac{1}{2}$ gallons of gasoline in your car's fuel tank.

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Look at **Example Set 1** to see how to change mixed numbers into improper fractions and improper fractions into mixed numbers.

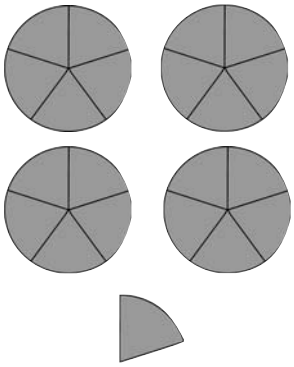
Example Set 1

Step 1 $4\frac{1}{5}$



Look at the mixed number, $4^{1}/_{5}$, as if it represented pies. You would see that four whole pies are each cut into 5 pieces and one piece remaining of another pie that has been cut into five equal pieces. If three friends came over and each of them wanted to eat one piece of pie with you, one of the four whole pies that have been cut in five equal pieces would have to be used.

Step 2 $3\frac{6}{5}$



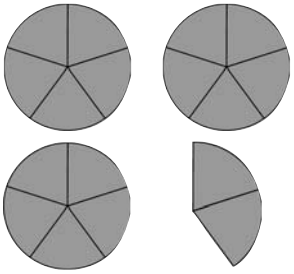
The mixed number now represents three whole pies and six pieces (numerator) of size five (denominator). Cutting one pie is necessary for you and your friends to eat one slice of pie each. An ***equivalent*** mathematical model for this statement is:

$$4\frac{1}{5}-\frac{4}{5}=3\frac{6}{5}-\frac{4}{5}$$

Looking at **Example Set 1**, Steps 1 and 2, you see four whole pies. You needed four smaller pieces of pie, but had only one, therefore, one of the whole pies had to be cut into equal pieces (in this case fifths or the whole cut into five equal slices). After cutting one pie into five equal slices, three whole pies remain. You now have six slices of size one-fifth, or $3^{6}/_{5}$. After serving yourself and your friends, the amount of pie remaining and the equation to represent that amount are shown in Step 3 on the next page.

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Step 3 $4\frac{1}{5}-\frac{4}{5}=3\frac{6}{5}-\frac{4}{5}=3\frac{6-4}{5}=3\frac{2}{5}$



Looking at the equation as it changes form, you can see that when four of the six pieces of pie were eaten, only two pieces remain. This process shows that to subtract fractions with the same or like denominators, the numbers in the numerators are subtracted. (See Rule 1 below.) Looking at the pie graph representation of the final answer, you can see the three whole pies and the two pieces remaining from the fourth pie.

RULE 1

The denominators of fractions must be alike or the same before addition or subtraction can be accomplished.

RULE 2

The subtraction of fractions with like denominators is accomplished by subtracting the

$\frac{5}{8}-\frac{4}{8}=\frac{5-4}{8}=\frac{1}{8}$ $\frac{9}{10}-\frac{6}{10}=\frac{9-6}{10}=\frac{3}{10}$ $\frac{8}{9}-\frac{7}{9}=\frac{8-7}{9}=\frac{1}{9}$

RULE 3

The addition of fractions with like denominators is accomplished by adding the numerators.

$\frac{5}{9}+\frac{3}{9}=\frac{5+3}{9}=\frac{8}{9}$ $\frac{3}{10}+\frac{6}{10}=\frac{3+6}{10}=\frac{9}{10}$ $\frac{9}{15}+\frac{4}{15}=\frac{9+4}{15}=\frac{13}{15}$

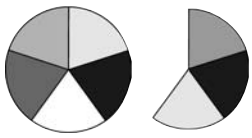
Rules 2 and 3 show that addition of fractions follows the same basic principle as subtraction. When the denominators of two or more fractions are the same, adding the numerators performs the addition of those fractions.

Remember, the definition of a proper fraction stated that the numerator is less than the denominator. With this definition in mind, look at **Example Set 2** and notice how the improper fraction is changed into a whole number combined with a proper fraction.

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Example Set 2

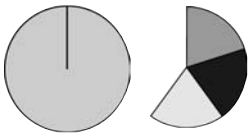
Step 1 $\frac{8}{5}$



$\frac{5}{5}+\frac{3}{5}$

This fraction contains eight pieces (numerator) of size five (denominator). Look at the pie graphs; five of the eight pieces are placed together in one pan, and three pieces remain in the second pan.

Step 2



$1+\frac{3}{5}$

Notice that the pie pan with five pieces now has become one whole pie. Remember the rule which states that if the numerator and denominator of a fraction are the same number, the fraction is written as *1*. ($^5/_5 = 1$)

Step 3 $\frac{8}{5}=1\frac{3}{5}$

The proper form of $^8/_5$ then is $1^3/_5$. The numerator of the fraction is less than the denominator. <u>All final answers containing fractions must be written as proper fractions</u>.

Demonstrate your understanding of mixed numbers by working the activities for this lesson.

Life Principle



"Clean never rubs off on dirty, but dirty always rubs off on clean."

—Dr. Charles Swindol

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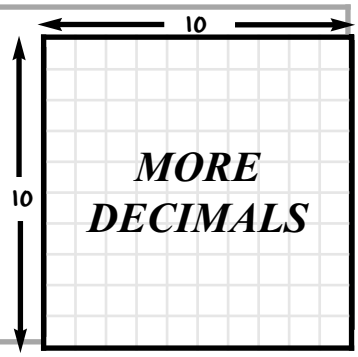
Lesson 7

Decimals

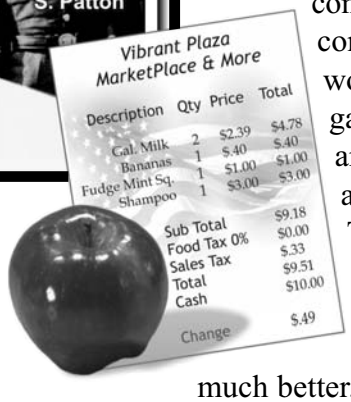
Chapter 2

Section 2

Lessons 6-10







Many people have said that history repeats itself. By studying history you can see parallels to the events in today's world. Professor K remembers that in World War II, the leaders of two opposing tank divisions (the United States led by General Patton and Germany's General Rommel) raced to a certain mountain pass in Africa. Each of these men knew from their study of history that whoever controlled that mountain pass would control the entire area. General Patton won the race to the pass and thereby gave the Allies ultimate control of the area. These two generals put a great amount of effort into what they did. Their job was to command tanks and soldiers, but the generals realized that knowing many other things would help them do the job at hand much better.

Professor K believes the same is true in your study of mathematics. Studying mathematics can help you in other areas of life, even if you do not become a soldier, mathematician or scientist. Just as history is said to repeat itself, mathematical principles and concepts learned previously are often repeated as you progress to new levels of mathematical skills. In this lesson Professor K will teach you more about decimals.

Decimals can be terminating or repeating. The decimal equivalents of the fractions in **Example Set 1** are ***terminating decimals*** because, when the denominator divides into the numerator, the quotient has no remainder, therefore the digits in the quotient stop or terminate.

VOCABULARY

Terminating decimals: decimals in which the digits stop

Indefinitely: going on and on without limits

Repeating decimal: a decimal in which a group of one or more digits repeats indefinitely

Repetend: the group of digits in a repeating decimal that repeats

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Example Set 1

Fraction	Decimal	Division
$\frac{1}{2}$	.5	$2\overline{)1.0}$ $-\ 1.0$ 0
$\frac{1}{4}$	.25	$4\overline{)1.00}$ $-\ 8$ 20 $-\ 20$ 0
$\frac{2}{5}$	.4	$5\overline{)2.0}$ $-\ 2.0$ 0
$\frac{3}{8}$	.375	$8\overline{)3.000}$ $-\ 2.4$ 60 $-\ 56$ 40 $-\ 40$ 0

Professor K wants you to notice that the divisions in **Example Set 1** show that the remainders are zero. Therefore the fractions can be represented by a terminating decimal.



DEFINITION: TERMINATING DECIMAL

A decimal that represents a fraction in which the denominator divides into the numerator evenly, or the quotient has a remainder of zero, is said to be a terminating decimal.



In **Example Set 2**, Professor K shows division in which the remainders will never become zero. In these examples, a group of one or more digits repeat ***indefinitely***, therefore, the decimal equivalents are said to be ***repeating decimals***.

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Example Set 2

Fraction	Decimal	Division
$\frac{1}{3}$	.3333... or $.\overline{3}$	$3\overline{)1.0000}$ -9 10 -9 10 -9 10 -9 1
$\frac{2}{3}$	.6666... or $.\overline{6}$	$3\overline{)2.0000}$ -1.8 20 -18 20 -18 20 -18 2
$\frac{101}{99}$	1.02020202... or $1.\overline{02}$	$99\overline{)101.000000}$ -99 2.00 -1.98 200 -198 200 -198 2



DEFINITION: REPEATING DECIMAL

A decimal that represents a fraction in which the denominator divides into the numerator and the quotient does not have a remainder of zero, and if one or more of the digits in the decimal portion of the quotient repeats indefinitely is said to be a repeating decimal.

The repeating portion of the decimal is signified by three dots (...) following the repeating digits, or a bar (‾) over the top of the repeating digit or digits.

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DEFINITION: REPETEND

A ***repetend*** is the group of digits of a decimal that repeats. The repetend can be one digit or a group of more than one digit.



Look at **Example Set 3**. Notice the digits that repeat in each decimal, then notice which digits are the repetend.

Example Set 3

Decimal	Repetend
.777... or $.\overline{7}$	7
.444... or $.\overline{4}$	4
1.123123... or $1.\overline{123}$	123

Demonstrate your understanding of terminating and repeating decimals by working the activities for this lesson.

Life Principle



"We learn from experience (history) that we never learn from experience."

—George Bernard Shaw (1856-1950)

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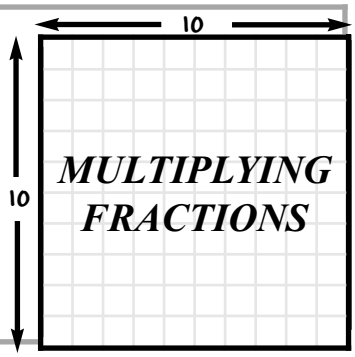
Basic Math Skills



Fractions

Lesson 8

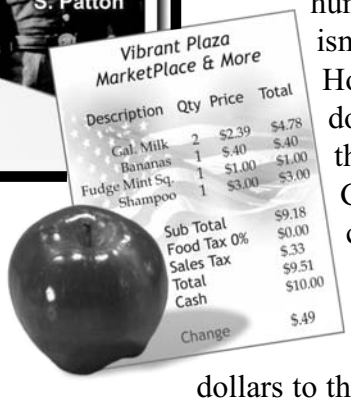
Chapter 2



Section 2

Lessons 6-10







Professor K

A shopping trip to the grocery store gave Professor K an opportunity to discuss sales taxes with his children. They had noticed that "good" food items were not taxed, but nonfood and ***"junk" food*** items were taxed. Professor K's children wanted to know the reason for sales taxes. Professor K explained a penny is one one-hundredth ($^1/_{100}$) of a dollar. A penny isn't much money all by itself. However, when this fractional part of a dollar is multiplied many times over, the total dollars can add up. Governments use this principle in the collection of sales taxes. The few pennies in sales tax charged on each purchase multiplied by the large number of purchases nets millions of dollars to the treasuries of state and local governments. This money is used to support various government agencies and programs. It is important to remember that all money paid by government agencies is collected from private business and consumers.

The process of *multiplying* two or more fractions together is similar to the addition of two or more fractions. When multiplying two or more fractions, the denominators do not have to be the same number as with addition or subtraction. Professor K shows in Rule 1 the process used to multiply two fractions together.



RULE 1

Multiplying numerators and denominators accomplishes multiplication of fractions.

VOCABULARY

"Junk" food: food containing little or no nutritional value; candy, soft drinks, chips

Factorization: the process of splitting a number into all its separate factors

Canceling: a term often used to describe the process of "dividing out" factors in a fraction

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$$\frac{1}{2}\times\frac{1}{3}=\frac{1\times1}{2\times3}=\frac{1}{6} \quad\Big|\quad \frac{2}{3}\times\frac{3}{4}=\frac{2\times3}{3\times4}=\frac{6}{12}$$



Professor K demonstrates that multiplying more than two fractions together is accomplished by extending Rule 1. Look at **Example Set 1** below to see how Professor K completed this process.

Example Set 1

a) $\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}=\frac{1\times1\times1}{2\times3\times4}=\frac{1}{6\times4}=\frac{1}{24}$

a) Multiplying the numerators of the three fractions together yields *1 × 1 × 1 = 1*. The product, *1*, is placed in the numerator of the final product. Multiplying the denominators together yields the expression: *2 × 3 × 4*. Then multiply *2 × 3 = 6*. Next multiply *6 × 4 = 24*. The *24* is placed in the denominator of the final product. The final product is $^{1}/_{24}$.

b) $\frac{1}{5}\times\frac{2}{5}\times\frac{3}{5}=\frac{1\times2\times3}{5\times5\times5}=\frac{2\times3}{25\times5}=\frac{6}{125}$

b) In this problem the numerators are multiplied together as follows: *1 × 2 = 2*, then *2 × 3 = 6*. The *6* is placed in the numerator of the final product. The denominators are multiplied together: *5 × 5 = 25*, then *25 × 5 = 125*. The *125* is placed in the denominator of the final product. The final product is $^{6}/_{125}$.

c) $\frac{2}{3}\times\frac{4}{5}\times\frac{1}{2}=\frac{2\times4\times1}{3\times5\times2}=\frac{8\times1}{15\times2}=\frac{8}{30}$

c) The numerators of this problem are multiplied together as follows: *2 × 4 = 8*, then *8 × 1 = 8*. The *8* is placed in the numerator of the final product. The denominators are multiplied together in the following manner: *3 × 5 = 15*, then *15 × 2 = 30*. Thirty is placed in the denominator of the final product. The final product is $^{8}/_{30}$.

Notice in **Example Set 1 b)**, even though the denominators are the same, they are still multiplied together to have a final product of $^{6}/_{125}$. Caution should be exercised not to confuse multiplication of such fractions with addition. Such a mistake might turn the fraction into $^{6}/_{5}$, if the numerators were multiplied, but not the denominators since they were the same number. The denominators must be multiplied together, as well as the numerators.

In **Example Set 1**, all of the products are, by definition, proper fractions (in which the numerator is less than the denominator). However, $^{8}/_{30}$ is not written in a form that is

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commonly accepted for the final answer for a fraction. The $^{8}/_{30}$ must be reduced. Reducing a fraction to its simplest form means "dividing out" all the common factors until the only factor common to both the numerator and denominator is one (1).

Example Set 2 shows how the numerator and denominator of $^{8}/_{30}$ are separated into the various factors and then "divided out."

Example Set 2

Step 1

$$\frac{8}{30}=\frac{1\times2\times4}{1\times2\times15}$$

The factors of the numerator, *8*, are *1*, *2*, 2, and 2 because 1 × 2 × 2 × 2 = 8. In the denominator, the initial factors of *30* are *1*, *2*, *3*, 5 because *1* × *2* × 3 × 5 = *30*. (See Step 1.)

Step 2

$$\frac{1\times2\times2\times2}{1\times2\times3\times5}=1\times1\times\frac{2\times2}{3\times5}$$

Notice that in the complete ***factorization*** of the numerator and denominator of the original fraction, the factors *1* and *2* are in both the numerator and denominator. Thus, $^{1}/_{1}$ = *1* and $^{2}/_{2}$ = *1*, therefore the new problem is shown in Step 2.

Step 3

$$\frac{2\times2}{3\times5}=\frac{4}{15}$$

In the numerator *2* × *2* = *4*, and the *4* is placed in the numerator of the final product. In the denominator, *3* × *5* = *15*, and the *15* is placed in the denominator of the final product. This makes the final product, $^{4}/_{15}$. We say that $^{8}/_{30}$ has now been reduced to lowest terms, meaning that no number besides one will divide into *4* and *15* evenly.

Division of the common factors is often referred to as ***canceling***. However, this implies that the factors somehow vanish mysteriously. This is not the case as is seen in **Example Set 2**. Instead of vanishing, the factors are changed into a fraction equal to one; $^{1}/_{1}$ = *1* and $^{2}/_{2}$ = *1*. (If the numerator and denominator are the same number, or the numerator and denominator are equal, then the value of the fraction is one, *1*.)

Apply what you have learned about multiplying and dividing fractions by working the activities for Lesson 8.

Life Principle



"The role of government is to do good and punish evil among the people served."

—Paul of Tarsus

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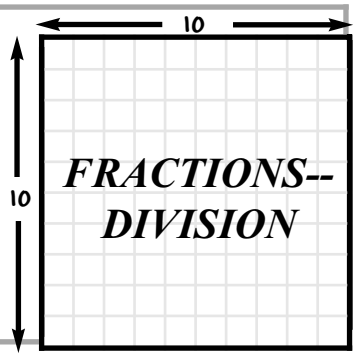
Basic Math Skills



Fractions

Lesson 9

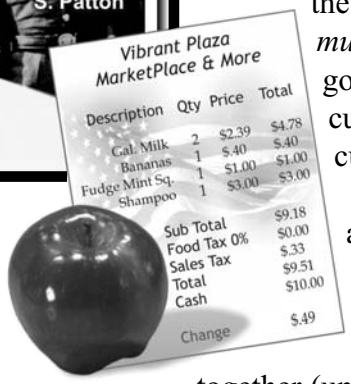
Chapter 2



Section 2

Lessons 6-10







Professor K

Professor K has thought often of the terrorist attacks on the Twin Towers on 9-11-2001. The enemies of the United States thought that fear and doubt would *divide* the people of this great country. However, the opposite was true. The devastating attack on two symbols of the free enterprise system proved only to unite the citizens of the United States to *multiply* their efforts toward a common goal of preserving the American culture based on traditional family and cultural values.

The division of fractions is accomplished in much the same way as simple multiplication of fractions. First the problem is division, but then the fractions are brought together (united) by multiplication. However, a process known as finding the ***reciprocal*** of a number or a fraction must be performed before this multiplication can take place.



Reciprocal: the multiplicative inverse of a number; fractional result obtained by inverting a fraction. Two numbers are reciprocals of each other if their product equals one.
Inverted: turned over or flipped

Principle



To find the reciprocal of a number, the number is ***inverted*** or flipped. (See **Example Set 1**) Professor K reminds you that any whole number can be written in fraction form with a 1 in the denominator. Therefore, the reciprocal of a whole number would have a 1 in the numerator and the whole number would be placed in the denominator. Professor K demonstrates this concept in **Example Set 1**.

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Example Set 1

	Whole number	Fraction	Reciprocal
a)	2	$\frac{2}{1}$	$\frac{1}{2}$
b)	25	$\frac{25}{1}$	$\frac{1}{25}$

	Whole number	Fraction	Reciprocal
c)	4	$\frac{4}{1}$	$\frac{1}{4}$
d)	1234	$\frac{1234}{1}$	$\frac{1}{1234}$

Professor K shows you how to find the reciprocals of fractions in **Example Set 2**.

Example Set 2

	Fraction	Reciprocal
a)	$\frac{2}{3}$	$\frac{3}{2}$
b)	$\frac{5}{6}$	$\frac{6}{5}$

	Fraction	Reciprocal
c)	$\frac{3}{4}$	$\frac{4}{3}$
d)	$\frac{3}{31}$	$\frac{31}{3}$

In **Example Sets 3 & 4,** Professor K takes the process of finding the reciprocal into a division problem containing fractions. You must follow the steps of Rule 1 as the division of fractions is demonstrated in the examples below.



RULE 1

To divide fractions, find the reciprocal (invert or flip) of the divisor, and multiply.
(See **Example Set 3**)



Example Set 3

Step 1 $\frac{2}{3} \div \frac{1}{8}$

Step 1 states the problem. Notice the divisor, $^1/_8$, to the right of the division operator symbol (÷).

Step 2 $\frac{1}{8} \rightarrow \frac{8}{1}$

The reciprocal of $^1/_8$ is found as the first major step to solve the problem. The reciprocal of $^1/_8$ is $^8/_1$. The → symbol means *changes to, implies, or becomes*. (See Step 2.)

Step 3 $\frac{2}{3} \times \frac{8}{1} = \frac{2 \times 8}{3 \times 1} = \frac{16}{3}$

The division problem now becomes a multiplication problem (Rule 1). The numerators and denominators are multiplied as shown, yielding a final product of $^{16}/_3$. (See Step 3.)

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Step 4	$\frac{16}{3} = 5\frac{1}{3}$	The final product is an improper fraction and must be changed into a mixed number and reduced to lowest terms to be written in proper form. Three divides into sixteen fives times with a remainder of one. The answer is 5, remainder 1. In fractions, the remainder (1) is placed over the divisor (3) in the quotient. Therefore, the answer is $5\frac{1}{3}$. (See Step 4.)

Example Set 4

Step 1	$\frac{3}{4} \div \frac{9}{8}$	Step 1 states the problem. Notice the divisor, $\frac{9}{8}$, to the right of the division operator symbol.
Step 2	$\frac{9}{8} \rightarrow \frac{8}{9}$	The reciprocal of $\frac{9}{8}$ is found as the first major step to solve the problem. The reciprocal of $\frac{9}{8}$ is $\frac{8}{9}$. Remember, the → symbol means *changes* to, *implies*, or *becomes*. (See Step 2.)
Step 3	$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9} = \frac{24}{36}$	The division problem now becomes a multiplication problem (Rule 1). The numerators and denominators are multiplied as shown yielding a final product of $\frac{24}{36}$. (See Step 3.)
Step 4	$\frac{24}{36} = \frac{2 \times 2 \times 2 \times 3}{2 \times 2 \times 3 \times 3}$	The final product is a proper fraction, which always must be reduced to lowest terms for final presentation. Factorization of the numerator and denominator is shown in Step 4.
Step 5	$\frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{3}}{\cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{3}} = \frac{2}{3}$	This yields the new problem in Step 5. The final answer of $\frac{2}{3}$ is produced by cancelling all like numbers across the divide line. Only the numerator "2" and the denominator "3" are left as the answer. (See Step 5.)

Professor K reminds you that any fractional answer must be presented in the proper reduced form as $5\frac{1}{3}$ in **Example Set 3** (Step 4), and $\frac{2}{3}$ in **Example Set 4** (Step 5).

Apply what you have learned about dividing fractions by working the activities for Lesson 9.



Life Principle

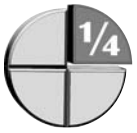
"In God we trust. United We Stand."

—New York City firemen after 9-11-2001

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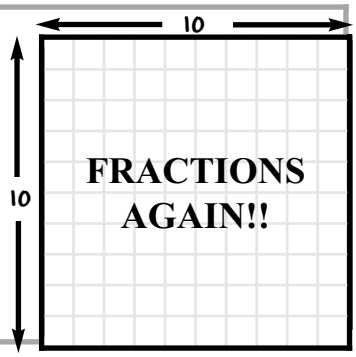
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Fractions

Lesson 10

Chapter 2



Section 2

Lessons 6-10

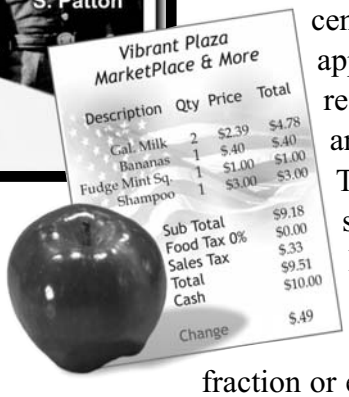




Professor K

A college professor once explained the mathematical principle used in this lesson to Professor K in a simplified manner. Professor K explains it to you as follows:

A man had two children. He went to a store and purchased an apple for each child. One apple cost 50 cents; the other cost 99 cents. When the man presented the apples to his children, the child who received the 50-cent apple was upset, and accused the father of being unfair. The child thought that the same size should have been given to him as to his ***sibling***.



A mathematical principle states, what is done to one part of a fraction or equation, must be done for the other part. The mathematician would not want one part of his mathematical problem to be treated unfairly or get upset because he didn't do exactly the same thing to both parts of the equation.

COMMON DENOMINATOR

You learned a fact in Lesson 6, Rule 2: placing the sum of the numerators of the fractions in the numerator of the answer and keeping the same denominator accomplishes the addition of fractions with like denominators.

Lesson 10 will show you how to find a ***common denominator,*** or more plainly stated, make the denominators of fractions the same when they are not the same to begin with in a problem. Another fact you must remember is found in the following principle.

VOCABULARY

Sibling: brother or sister
Common denominator: a number that two or more denominators of separate fractions can be changed into so that the resulting fractions have the same denominator and therefore can be added or subtracted.
Lowest common denominator (LCD): the smallest number that the denominators of two or more fractions can divide evenly, or are factors of.

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Principle of Multiplying a Fraction by 1



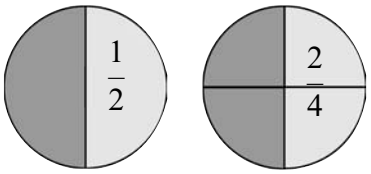
Multiplying the numerator and denominator of a fraction by the same number does not change the value of the fraction; but instead, it changes the fraction to an equivalent fraction of the same value.

The principle above is the same as multiplying a fraction by 1, because a fraction with an equal numerator and denominator has a value equal to one. When the same number multiplies the numerator and denominator, you are actually multiplying the fraction by one, therefore the fraction retains its original value (as we learned earlier in The Multiplicative Property of One). Look at **Example Set 1** as Professor K clarifies this procedure for you.

Example Set 1

Step 1 $\frac{1}{2}\times\frac{2}{2}=\frac{1\times2}{2\times2}=\frac{2}{4}$

The numerator and denominator of the fraction $\frac{1}{2}$ are each multiplied by 2. The product is $\frac{2}{4}$. (See Step 1.)



Looking at the pie graph, you can see that $\frac{2}{4}$ is equal to $\frac{1}{2}$.

Step 2 $\frac{1}{2}\times1=\frac{1}{2}\times\frac{1}{1}=\frac{1\times1}{2\times1}$

Step 2 shows the multiplication of one half times one to show that the answer remains the same as Step 1.

Remember, multiplying a number by a fraction in which the numerator and denominator are equal, is the same as multiplying the number by one, as Professor K demonstrated in **Example Set 1**.

Rule 1

If a certain number multiplies the numerator of a fraction, the same number must also multiply the denominator to maintain the same value of the original fraction.

$$\frac{1}{2}=\frac{1}{2}\times\frac{2}{2}=\frac{1\times2}{2\times2}=\frac{2}{4} \qquad \frac{1}{3}=\frac{1}{3}\times\frac{3}{3}=\frac{1\times3}{3\times3}=\frac{3}{9}$$

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Addition of two fractions requires that the denominators of the fractions be equal. When the denominators of two fractions are not equal, the fractions cannot be added until the denominators are made equal by finding a common denominator. This process makes the denominators equal and only then can the fractions be added.

DEFINITION

A ***common denominator*** is a number that two or more denominators of separate fractions can be changed into so that the resulting fractions have the same denominator and therefore can be added or subtracted.



In **Example Set 2**, Professor K demonstrates the process of finding a common denominator.

Example Set 2

Consider the two fractions, $^1/_2$ and $^2/_3$. The denominators are *2* and *3*. To find the common denominator of the two fractions the following question must be asked. What is the smallest number (least common denominator) that contains both *2* and *3* as factors? In this case, *6* is that number ($2 \times 3 = 6$).

The next step to find the least common denominator for the fractions, $^1/_2$ and $^2/_3$, is to determine what number times each denominator, *2* and *3*, will produce a denominator of *6*.

The denominator, *2*, of $^1/_2$, must be multiplied by three to obtain a product of 6. The denominator, *3*, of $^2/_3$, must be multiplied by two to obtain a product of *6*. Remember, the same operation must be performed on one part of a fraction as on the other part of a fraction.

The numerator and denominator of $^1/_2$ must be multiplied by 3 to change the denominator to 6. The numerator and denominator of $^2/_3$ must be multiplied by 2 to change the denominator to 6.

$$\frac{1}{2}\times\frac{3}{3}=\frac{1\times3}{2\times3}=\frac{3}{6} \qquad \frac{2}{3}\times\frac{2}{2}=\frac{2\times2}{3\times2}=\frac{4}{6}$$

This process has made the denominators the same, *6*. Now the two fractions can be added or subtracted. Notice that the final fractions are not reduced, because this is only a necessary step so the fractions can be added. Only the final answer must be presented in reduced form. Once the fractions have the same or common denominator, the fractions can be added or subtracted.

Example Set 3 shows the addition and subtraction of the two fractions, $^1/_2$ and $^2/_3$.

Example Set 3

Addition

Step 1 $\frac{1}{2}\times\frac{3}{3}=\frac{1\times3}{2\times3}=\frac{3}{6}$ and $\frac{2}{3}\times\frac{2}{2}=\frac{2\times2}{3\times2}=\frac{4}{6}$

In Step 1 the multiplications to find the common denominators in **Example Set 2** are restated so the fractions $^1/_2$ and $^2/_3$ can be added and subtracted.

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Step 2 $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}=\frac{3+4}{6}=\frac{7}{6}=1\frac{1}{6}$

Replace the original fractions, $^1/_2$ and $^2/_3$, with their fractional equivalents, $^3/_6$ and $^4/_6$, then add the numerators. The sum is the improper fraction, $^7/_6$. The *6* will divide into *7* one time with a remainder of one. This changes the improper fraction into the mixed number $1^1/_6$.

Subtraction

Step 3 $\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}=\frac{4-3}{6}=\frac{1}{6}$

Step 3 shows the subtraction of $^1/_2$ from $^2/_3$. The difference, $^1/_6$, is in proper form for presentation of the final answer.

Practice finding the common denominator by working the activities for this lesson.

Life Principle

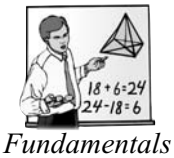


"Working toward a common goal brings people together."

—Amy Luna

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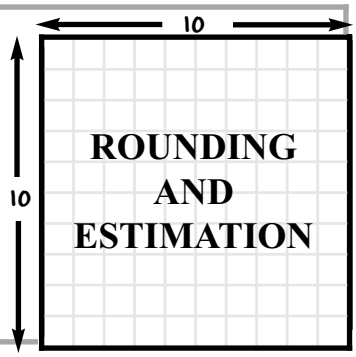
Basic Math Skills



Lesson 11

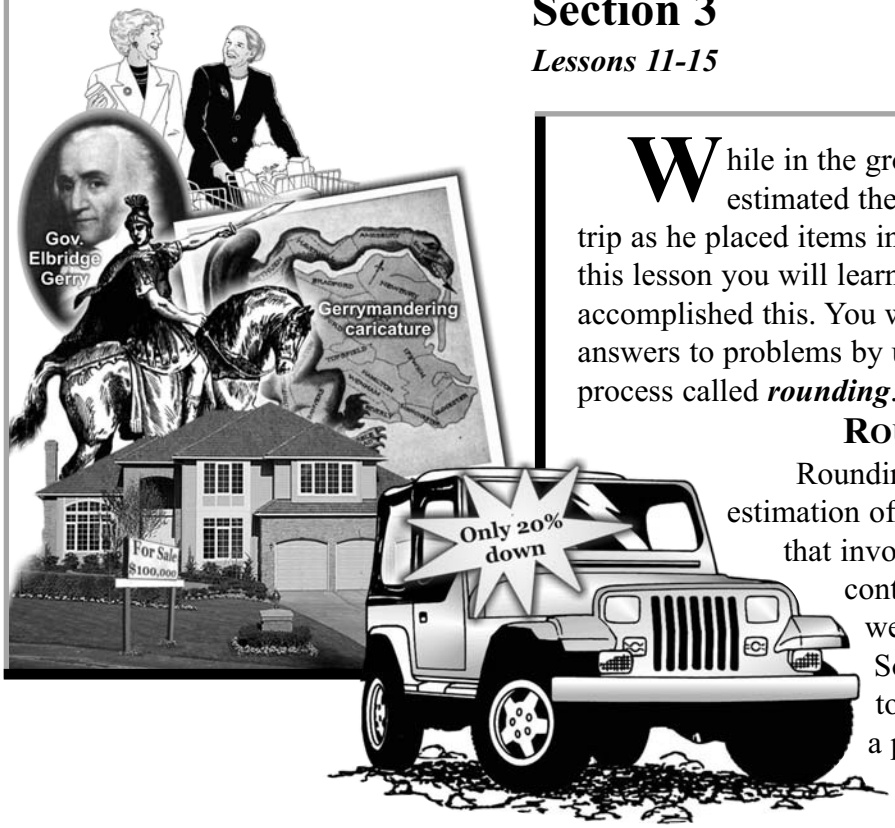
Fundamentals

Chapter 2



Section 3

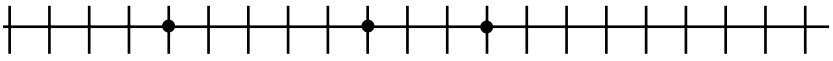
Lessons 11-15



While in the grocery store, Professor K estimated the total cost of his shopping trip as he placed items into his shopping cart. In this lesson you will learn how Professor K accomplished this. You will learn about ***estimating*** answers to problems by using a mathematical process called ***rounding***.

ROUNDING

Rounding allows you to make an estimation of the answer to a problem that involves whole numbers containing several digits, as well as fractions or decimals. Sometimes you do not have to know an exact answer to a problem. The distance around the earth or the number of people living in the United States is usually only ***approximations*** or estimates of the exact answer. To visualize rounding as Professor K does, look at the number line below.



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

When rounding numbers you round "up" or "down" to the closest number that you desire. If the instructions were to round *4*, *9* and *12* to the nearest tens place, then you can look at the number line to visualize where to round a number (*4*, *9*, *12* represented by the dots).

- *4* rounds "down" to *0*, because *4* is closer to zero, than to 10
- *9* rounds "up" to 10, because *9* is closer to 10 than to zero.
- *12* rounds "down" to 10, because *12* is closer to 10 than to the next *ten* up the line, which is 20.

VOCABULARY

Estimating: making a logical calculated guess; approximating
Rounding: a mathematical procedure used to make estimations of an outcome
Approximations: almost exact but not completely so
Calculations: figuring solutions, estimating, computing

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Example Set 1 shows how rounding works for problems involving these numbers.

Example Set 1

a) 4 × 9

a) In this problem, *4* × *9* = 36. This is relatively easy; however, if you wanted only an estimate of the final answer, you could round the *9* to 10 and the problem becomes, 4 × 10 = 40. Notice that the rounding of *4* to the nearest tens place would produce a zero for the answer and would have no meaning for this problem.

b) 9 × 12

b) In this problem, *9* × *12* = 108. If an estimate is all you need, round the *9* to 10 and the *12* to 10. The problem becomes 10 × 10 = 100.

c) 4 × 12

c) In this problem *4* × *12* = 48. An estimate in which *4* is left as it is, and *12* is rounded to 10, produces the problem 4 × 10 = 40. Notice that if the rounding of the *4* is "fine tuned", the *4* could round to 5. The problem would become 5 × 10 = 50. This estimate is closer to the correct answer.

Numbers can be rounded to any place that is needed to obtain the accuracy desired. **Example Set 2** gives a number and then rounds that number to the indicated place value.

Example Set 2

The number before rounding: 5,627,189



Round to the nearest	**Rounded number**
tens place	The *9* in the *ones* place of the original number is rounded "up" to 1 group of ten and added to the *8* in the *tens* place, *8* + 1 = 9. Rounded to the *tens* place, the number becomes 5,627,190.
hundreds place	*189* in the original number is closer to 200 than 100, so 189 is rounded "up" to 200. Thus rounded to the *hundreds* place, the number becomes 5,627,200.
thousands place	*7,189* in the original number is rounded "down" to 7,000 because 7,000 is the nearest thousand to 7,189. Thus rounded to the *thousands* place the number becomes 5,627,000.
ten thousands place	*27,189* in the original number is rounded "up" to 30,000 because 30,000 is the nearest ten thousand to *27,189*. Therefore, rounded to the *ten thousands* place, the number becomes 5,630,000.

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hundred thousands place	*627,189* in the original number is rounded "down" to 600,000 because *627,189* is closer to 600,000 than to 700,000. Rounded to the *hundred thousands* place, the number becomes 5,600,000.
millions place	*5,627,189* is closer to 6 million than to 5 million, so the number is rounded to 6,000,000.



RULE 1



Determine the place value to be rounded. Look at the digit to the right of the place value to which you want to round the number.
If the digit to the right is 4 or less, round "down", to keep the same value in the place being rounded. If the value to the right is 5 or more, round "up"; add 1 to the value of the next digit in the place being rounded to.

9 is rounded up to become 10

Rounding of decimals is especially useful when dealing with money. Consider the cost of the list of items below placed in Professor K's shopping cart at the grocery store.

Actual cost	Item	Estimated Cost
$2.15	Milk	$2.00
$ .98	Bread	$1.00
$13.57	Laundry Detergent	$14.00
$ .44	Can of vegetables	$ .50
$ .65	Can of soup	$1.00
$1.89	Bag of apples	$2.00
$5.25	Trash Bags	$5.00
?????	Total bill	$25.50

Notice how Professor K estimated the cost for each item by rounding "up" or "down" according to **Rule 1**. Also notice that Professor K rounded across the decimal points in all but one case (the can of vegetables). Estimations such as this are very useful when shopping and determining whether you will have enough money when you get to the checkout register. With practice, ***calculations*** such as this can be accomplished in your head as Professor K does. Adding the actual cost reveals that the actual total of the merchandise listed is $24.93.

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Rounding in this instance was very accurate. Note: when estimating, unless otherwise instructed, round off numbers to make them easy for you to calculate and estimate the answer. Regarding the list of grocery items, addition was simplified when the actual costs were rounded to the dollars place.

Professor K shows that the difference of two numbers can also be estimated by the use of the principle of rounding as shown in **Example Set 3**.

Example Set 3

Actual	Rounded to nearest dollar	Actual	Rounded to nearest 10 dollars
\$475.43	\$475.00	\$137.75	\$140.00
− \$325.65	− \$326.00	− \$28.23	− \$30.00
\$149.78	\$149.00	\$109.52	\$110.00

ROUNDING (MULTIPLICATION)

Estimating the product of the multiplication of two numbers can be accomplished by the use of rounding. Professor K finds this to be very helpful for calculations accomplished in his head. Professor K demonstrates this process in **Example Set 4**.

Example Set 4

Actual	Rounded to nearest 10s	Actual	Rounded to nearest 100s
345	350	486	500
× 38	× 40	× 23	× 20
13,110	14,000	11,178	10,000

Rounding before performing a mathematical operation (addition, subtraction, multiplication, division) allows you to obtain an estimate of the actual answer by making the calculation easier. Sometimes this allows you to make the calculation of the estimated answer in your head. You can then check the estimated answer against the actual answer to be more certain that the actual answer is correct. Even though calculations are often completed using actual numbers (not rounded), the answer is rounded to a specific place value. Professor K demonstrates this concept in **Example Set 5**.



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Example Set 5

Actual	Rounded to nearest 10s	Actual	Rounded to nearest 1000s
$\begin{array}{r} 257 \\ +\ 56 \\ \hline 313 \end{array}$	$313 \rightarrow 310$	$\begin{array}{r} 123 \\ \times\ 13 \\ \hline 1{,}599 \end{array}$	$1{,}599 \rightarrow 1{,}600$

($\rightarrow$ means changes to, becomes, or implies)

Repeating decimals are usually rounded off as Professor K shows in **Example Set 6**.

Example Set 6

Actual	Rounded to nearest 100ths	Actual	Rounded to nearest 1000ths
1.6666	1.67	4.3333	4.333

Professor K encourages you to complete the activity questions to demonstrate your knowledge of rounding and estimating.

LIFE PRINCIPLE



"Take care of the pennies and the dollars will take care of themselves."

—Daniel Maddelena

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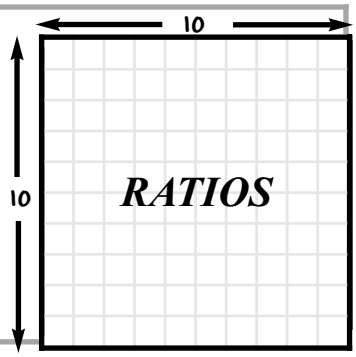
Basic Math Skills



Fractions

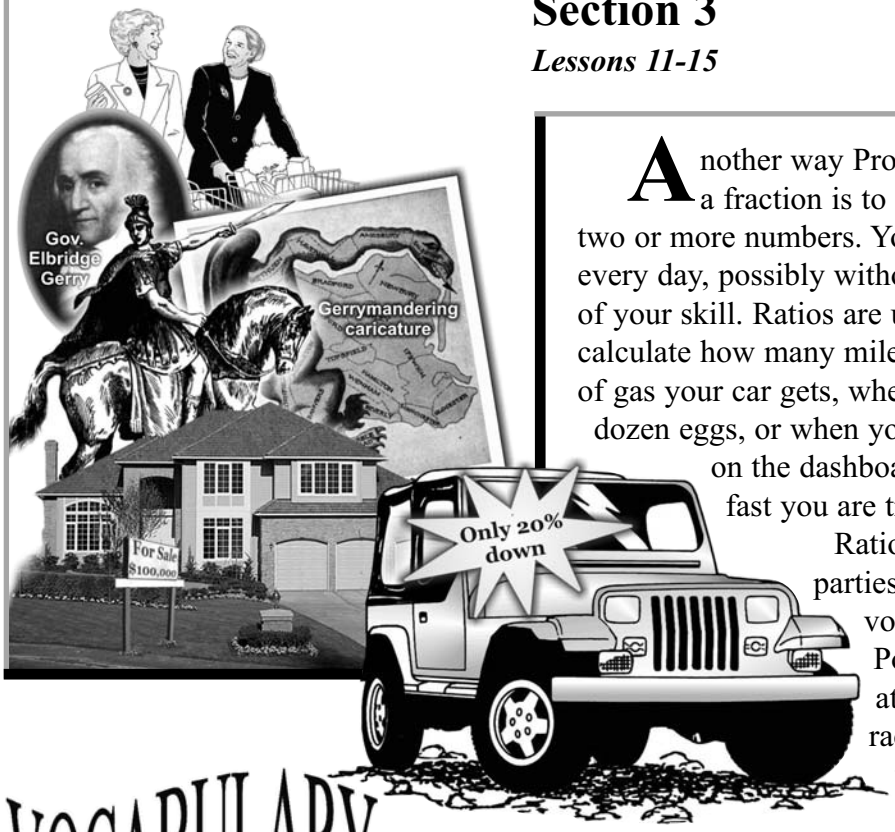
Lesson 12

Chapter 2



Section 3

Lessons 11-15





Professor K

Another way Professor K expresses a fraction is to find the ***ratio*** of two or more numbers. You use ratios every day, possibly without being aware of your skill. Ratios are used when you calculate how many miles to the gallon of gas your car gets, when you price a dozen eggs, or when you look at the ***speedometer*** on the dashboard of your car to see how fast you are traveling (miles per hour).

Ratios are important to political parties in the ***apportioning*** of voting districts for elections. Political parties look closely at the ratios of one electoral race to another race, men to women, young to old. This close look usually determines how a political party will conduct its affairs or set its agenda. Political parties sometimes use what is called "gerrymandering" to make sure certain voters are confined to particular voting districts. The word, ***"gerrymander"*** comes from the name of the governor of Massachusetts in 1812, Gov. Elbridge Gerry, combined with the word, *salamander*. A drawing of the district set up by Gov. Gerry was long, slender, and curvy. The voting district looked something like a salamander, when a head, legs, and claws were added to the drawing. Thus came the term, "gerrymander." Gov. Gerry had mapped out a district where the ratio of the numbers of one party's members to another was substantially unfair. This act of "gerrymandering" assured Gerry's political party at least one win in the upcoming election.

History provides much evidence of problems regarding civil rights issues. "Gerrymandering" was very prevalent after the Civil War of 1860 - 1865. The most notable case of

VOCABULARY

Ratio: a comparison of two numbers
Speedometer: instrument on the dashboard of a vehicle for determining at what speed a vehicle is traveling
Apportioning: dividing or assigning according to a plan, especially money or voting districts
Gerrymander: to divide voting districts so as to give advantage to a particular political party or group
Proportions: equation of two ratios separated by an equal sign

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extreme "gerrymandering" occurred in one congressional district in which the Black vote was heavily concentrated. Instead of the district following county lines, roads, rivers, or having a standard shape, the district was 300 miles long and 20 miles wide.

Today, when voting district lines are redrawn because of population shifts, citizens' groups watch very carefully for "gerrymandering" in an attempt to keep elections fair to all voters.

Definition: Ratio



A ratio is a comparison of two or more numbers. Ratios are written in fraction form with a fraction bar or with a colon between the two numbers being compared.



$\frac{1}{2}$ or $1:2$ $\quad$ $\frac{1}{3}$ or $1:3$ $\quad$ $\frac{2}{9}$ or $2:9$ $\quad$ $\frac{25}{35}$ or $25:35$

A ratio stated in words is, "the ratio of a number to another number." Therefore, $1:2$ in words is, "the ratio of one to two"; $2:9$ in words is, "the ratio of two to nine."

Professor K points out that the final answer to a ratio problem must be written in reduced or simplified form when written in fraction or colon form. **(Remember, → means changes to, becomes, or implies.)**

$$\frac{2}{4} \rightarrow \frac{1}{2} \qquad \frac{3}{9} \rightarrow \frac{1}{3}$$

$$2:4 \rightarrow 1:2 \qquad 3:9 \rightarrow 1:3$$

Consider the number of boys to girls in a math class at school *A*, and the number of boys to girls in a math class at school *B*. Look at **Table 12.1**.

Table 12.1

School	Boys	Girls	Total
A	8	16	24
B	4	8	12

The ratio of the number of boys to girls in a math class in school *A* is 8:16. Written in fraction form the ratio is $^{8}/_{16}$. Simplifying this fraction yields, $^{1}/_{2}$, or one boy for every two girls. The ratio of the number of boys to girls in a math class at school *B* is 4:8. This ratio written as a fraction is $^{4}/_{8}$. Reducing, $^{4}/_{8}$, yields $^{1}/_{2}$, or one boy to every two girls. When the fractions representing the ratios of boys to girls in a math class at schools *A* and *B* are reduced, the resulting fractions of $^{1}/_{2}$ are equal. Therefore the ratios of boys to girls in math classes at schools *A* and *B* are equal. These ratios are also said to be proportional.

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Definition: Proportion

A ***proportion*** is an equation of two ratios separated by an equals sign. Two ratios are proportional if the fractions that represent those ratios are equal.

$$\frac{5}{10}=\frac{50}{100} \qquad \frac{3}{6}=\frac{6}{12} \qquad \frac{2}{5}=\frac{20}{50} \qquad \frac{250}{1000}=\frac{1}{4}$$

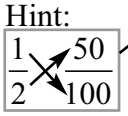
The ratios above that are separated by an equals sign are proportional.

Rule

An equation represents a proportion if the products of the cross-multiplications are equal.



Hint:



$\frac{1}{2}=\frac{50}{100} \rightarrow 1\times 100=2\times 50 \rightarrow 100=100$ ratios are proportional

$\frac{2}{6}=\frac{6}{12} \rightarrow 2\times 12=6\times 6 \rightarrow 24\neq 36$ ratios are not proportional

Example Set 1

Consider the information about schools *A* and *B* in **Table 12.1**. Another school, C, has the same ratio of boys to girls as do schools *A* and *B*. If school C has 30 girls in a Math class, how many boys should you expect to see?

Step 1 Set up a proportion using information from **Table 12.1** and what is known about school C. Remember, for ratios to be the same they must be equal. Notice the proportion below. You must find the number that replaces the ? to make the equation a proportion.

Ratio from school *B*, $^{4}/_{8} = {}^{?}/_{30}$ (ratio from school C).

Step 2 $\frac{4}{8}=\frac{1}{2}$ (reduced to lowest terms)

Step 3 15 must replace the *?* above the fraction bar in $^{?}/_{30}$ so the fraction will reduce to $^{1}/_{2}$.

$$\frac{15}{30}=\frac{1}{2}$$

Step 4 Since both $^{4}/_{8}$ and $^{15}/_{30}$ equal $^{1}/_{2}$, school C would have *15* boys and *30* girls in math class.

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Example Set 2

Consider the information about schools *A* and *B* in **Table 12.1**. Look at the total number of students in each math class. A ratio of boys to the total number of students and a ratio of girls to the total number of students can also be calculated.

School	Boys to total # of students	Girls to total # of students
A	$8:24=\frac{8}{24}=\frac{1}{3}$	$16:24=\frac{16}{24}=\frac{2}{3}$
B	$4:12=\frac{4}{12}=\frac{1}{3}$	$8:12=\frac{8}{12}=\frac{2}{3}$

Notice that the ratios of the boys to the total of both schools are equivalent, as well as the ratios of the girls to the total of both schools.

Demonstrate your understanding of proportions by working the activities for this lesson.

Life Principle



"Patience is the companion of wisdom."

—St. Augustine (354 - 430)

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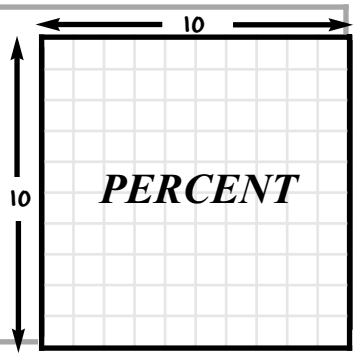
Basic Math Skills



Lesson 13

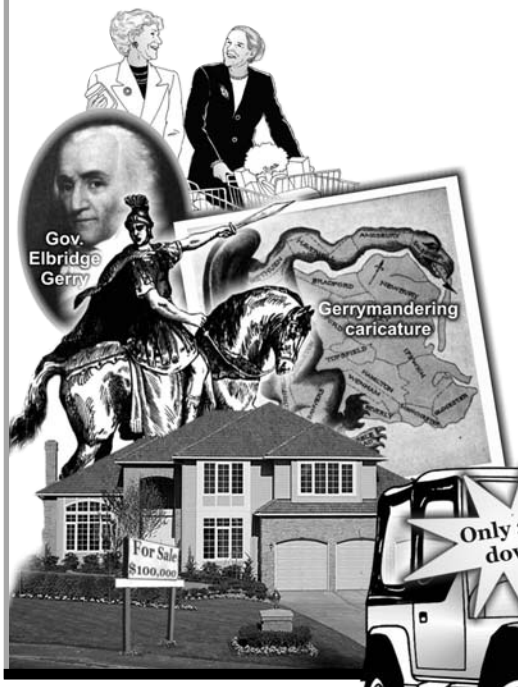
Percent

Chapter 2



Section 3

Lessons 11-15







Professor K

The concept of ratio and proportion is useful in another mathematical tool known as ***percent***. Professor K studied Latin in high school many years ago and knows that the word, percent, is derived from the Latin language. The Latin language is sometimes referred to as a "dead language" because Latin is not written or spoken as a national language in the world today. Even though Latin is not used in the average citizen's vocabulary, a large number of words in many languages have their ***origin*** from Latin words.

Cent is a portion of the word, *percent*. *Cent* comes from the Latin word, *centi-*, which means *one hundred*. Percent means "for every hundred." In ancient Rome, the commander of one hundred Roman soldiers was called a *centurion*. ***Eras*** of time can be blocked off into centuries, or periods of one hundred years. In U.S. ***currency***, another name for the penny is *cent* (100 cents = $1.00). The Mexican "penny" is known as a ***centavo*** (100 centavos = 1 ***Peso***).

Banks and governments use *percent* everyday. Banks charge interest on loans, and pay interest on savings accounts or CDs (***certificates of deposit***). Governments require taxes to be paid by the citizens, based on a certain percent of income (income tax) or purchases (sales tax). Department stores mark-up their merchandise or have sales based on the percent of the cost of the items.

VOCABULARY

Percent: from the Latin word *per centum*; per one hundred; for every one hundred
Origin: beginning; starting place
Eras: distinctive periods of time, anywhere from a few years to centuries
Currency: monetary method of exchange of goods and services in a country; money
Centavo: Mexican penny
Peso: 100 centavos; the currency of Mexico and several other countries

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Definition: Percent



Percent is a mathematical term used to describe a whole object cut into one hundred pieces. Percent is a ***standardized*** method of comparison using 100 as the standard.



$$n\% = \frac{n}{100} \qquad 20\% = \frac{20}{100} \qquad 33\% = \frac{33}{100}$$

In the examples of percent above, the "*n*" represents a number.

Changing Percents to Fractions

Professor K provides some examples of percents and their fractional equivalents in **Table 13.1**.

Table 13.1

Percent	Fraction	Percent	Fraction
200%	$\frac{200}{100} = 2$	50%	$\frac{50}{100} = \frac{1}{2}$
75%	$\frac{75}{100} = \frac{3}{4}$	25%	$\frac{25}{100} = \frac{1}{4}$
12.5%	$\frac{12.5}{100} = \frac{1}{8}$	66.7%	$\frac{66.7}{100} = \frac{2}{3}$
100%	$\frac{100}{100} = 1$	125%	$\frac{125}{100} = 1\frac{1}{4}$
17%	$\frac{17}{100}$	10%	$\frac{10}{100} = \frac{1}{10}$

Professor K wants you to notice that in **Table 13.1**, the *%* symbol signifies that the number to the left is placed in the numerator of the fraction and the denominator becomes *100*. Notice also that once the fraction is found, it is then reduced, if possible, to its simplest form. 100% of anything is all of it. In science, if something occurs 100% of the time, then it occurs all the time.

Changing Percents to Decimals

Example Set 1 on the next page shows how to change percents to fractions and then to decimals. Notice the placement of the decimal point in the decimal representation compared to the original number in the numerator.



VOCABULARY

Certificates of deposit: savings instrument available at banks and other lending institutions
Standardized: set formula or pattern

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Example Set 1

a) $50\% = \frac{50}{100} = .5$ $\quad .50. \rightarrow .5$

b) $75\% = \frac{75}{100} = .75$ $\quad .75. \rightarrow .75$

c) $66.7\% \approx \frac{67}{100} = .67$ $\quad .67. \rightarrow .67$

d) $13\% = \frac{13}{100} = .13$ $\quad .13. \rightarrow .13$

e) $125\% = \frac{125}{100} = 1.25$ $\quad 1.25. \rightarrow 1.25$

The decimal point (whether written or not) of the numbers in the numerator of each fraction is moved two places to the *left* and the denominator is dropped to obtain the decimal representation for each percent. Another way to approach changing percents to decimals is to divide the number by 100 and drop the % symbol.



(Remember, → means *changes to, becomes,* **or** *implies*.**)**

CHANGING DECIMALS TO PERCENTS

Decimals are changed to percents by multiplying the decimal times 100, or by moving the decimal point two places to the *right* of its original position. Place the resulting product in the numerator of a fraction with 100 as the denominator. For the representation of the decimal in percent form, place the numerator to the left of the % symbol and drop the 100 in the denominator. Look at **Example Set 2** for understanding of this procedure.

Example Set 2

a) $.56 = \frac{56}{100} = 56\%$ $\quad .56 \rightarrow 56\%$

b) $.25 = \frac{25}{100} = 25\%$ $\quad .25 \rightarrow 25\%$

c) $.37 = \frac{37}{100} = 37\%$ $\quad .37 \rightarrow 37\%$

d) $1.75 = \frac{175}{100} = 175\%$ $\quad 1.75 \rightarrow 175\%$

e) $.024 = \frac{2.4}{100} = 2.4\%$ $\quad .024 \rightarrow 2.4\%$

Notice that the decimal points of each of the decimals are moved two places to the right and placed in the numerator of a fraction containing a denominator of *100*. As you remember from the definition of percent, place the *%* symbol beside the number and drop the denominator of 100. Another way to approach changing decimals to percents is to multiply the number by 100 and add the % symbol. An easy way to remember this rule is to let the two 0s in the % symbol remind you to move two places to the right.

Professor K wants to check your understanding of percents by having you work the activities for this lesson.

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Life Principle



"You can fool some of the people all of the time, and all of the people some of the time; but you can't fool all of the people all of the time."

—Abraham Lincoln, U.S. President

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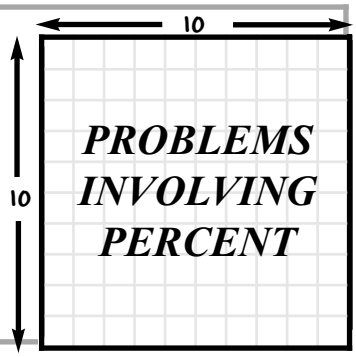
Basic Math Skills



Lesson 14

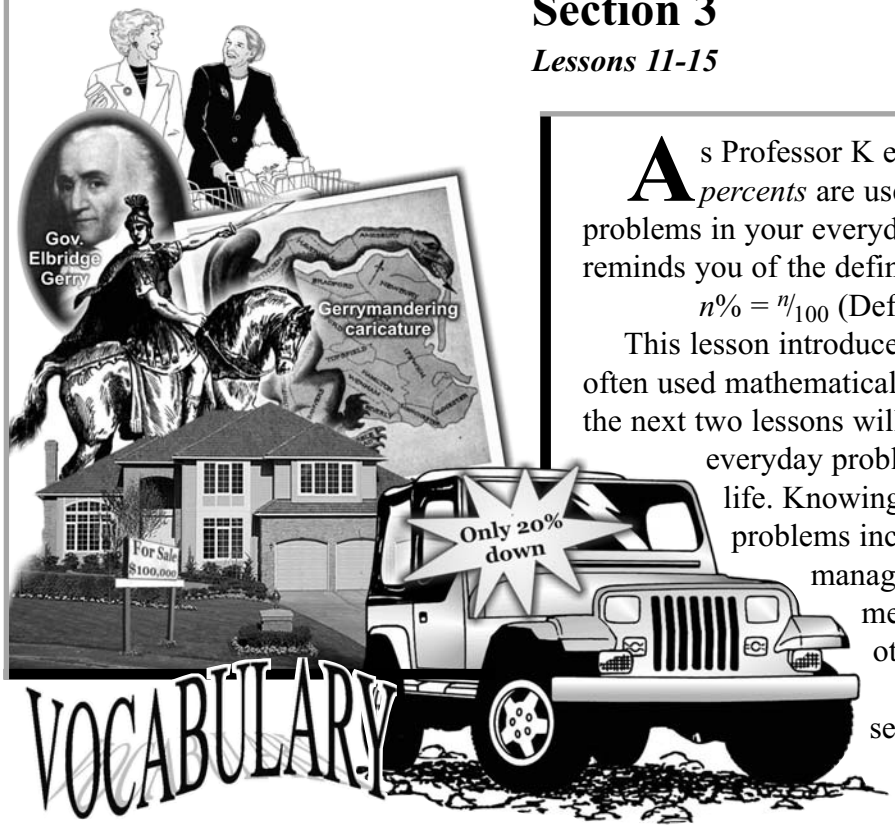
Percent

Chapter 2



Section 3

Lessons 11-15



As Professor K explained in Lesson 13, *percents* are used in many different problems in your everyday life. Professor K reminds you of the definition of *percent*.

$n\% = {}^{n}/_{100}$ (Definition of percent)

This lesson introduces you to three of the most often used mathematical skills. What you learn in the next two lessons will equip you to solve everyday problems throughout your adult life. Knowing how to work percent problems increases your ability to manage your bank account, plan meals, purchase a car and do other tasks.

Professor K knows that several problems are encountered when dealing with percents. These problems take three forms:

1. Finding the percent of a number.
2. Finding a number when a percent of that number is known.
3. Finding what percent one number is of another.

You must use what you have learned about changing percents to fractions and decimals, as well as remembering how to work with ratios and ***proportions*** from Lesson 12. Professor K will show you in **Example Set 1** how to find the percent of a number.

VOCABULARY

Unknown quantity: in math, the part of an equation signified by a letter (*n*, *x*, etc.) or question mark: the part of a math problem yet to be discovered or solved
lhs: left hand side of an equation
rhs: right hand side of an equation
Invest: to use money to bring about something profitable
Interest: a percentage of money earned on an investment (or paid for the privilege of borrowing money)

Example Set 1

Professor K wanted to purchase a new sports car advertised for $24,500.00. The finance company required a down payment of 20%. How much money was Professor K required to have as a down payment? Look at Step 1.

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Step 1 Set up a proportion using information from the word problem. Remember, for ratios to be the same they must be equal. Notice the proportion below where d is the ***unknown quantity*** or amount of the down payment.

$$\frac{d}{\$24,500} = \frac{20}{100}$$

You must find a number to replace the d to make the equation a proportion.

Step 2 You must cross multiply to find the answer. Therefore, the following equation results:

$$\frac{d}{\$24,500} = \frac{20}{100} \rightarrow d \times 100 = 20 \times 24,500$$

Step 3 You must isolate (set apart) the d on the left hand side of the equation (***lhs***) by dividing both sides of the equation by 100. (Just as with numerators and denominators of fractions, what you do to one side of the equal sign, you must do the same to the other side.)

$$\frac{d \times 100}{100} = \frac{20 \times 24,500}{100} \rightarrow d = \frac{20 \times 24,500}{100}$$

Notice that on the lhs, $d \times \frac{100}{100} = d \times 1 = d$ (Notice that 100 ÷ 100 = 1)

Step 4 Solve the right hand side of the equation (***rhs***).

$$\frac{20 \times 24,500}{100} = \frac{490,000}{100} = 4,900$$

(Notice that 490,000 ÷ 100 = 4,900)

This means that d = 4,900 and therefore, the required down payment is $4,900.00.

Professor K explains how to work an investment problem in **Example Set 2**.

Example Set 2

Mandy, Professor K's niece, worked at a summer job and ***invested*** the money she earned in a mutual fund at 15% ***interest***. At the end of one year Mandy had earned $375.00 in interest. You may calculate the amount of money Mandy invested by setting up an equation with the 15% and $375.00.

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Step 1 Set up a proportion using information from the word problem.

$$\begin{matrix}\text{(interest)}\\ \text{(money invested)}\end{matrix}\ \frac{\$375.00}{m}=\frac{15}{100}\ \text{(percent)}$$

In this problem the unknown number, *m*, is placed in the denominator of the lhs. The value of the interest is known, *$375.00*, and the fractional representation of the percent is known, $^{15}/_{100}$.

Step 2 You must cross multiply to find the answer. Therefore, the following equation results:

$$\frac{\$375.00}{m}=\frac{15}{100}\rightarrow m\times 15=\$375.00\times 100$$

Step 3 The *m* must be isolated on the lhs. *15* must divide both sides of the equation to accomplish the isolation of the *m*.



$$\frac{m\times 15}{15}=\frac{\$375.00\times 100}{15}\rightarrow m=\frac{\$37{,}500.00}{15}$$

Step 4 Simplify the rhs (divide *$37,500.00* by *15*) to find that *m* = 2,500. Therefore, Mandy invested $2,500.00 in the mutual fund.

Sometimes you must find what percent one number is of another number. In **Example Set 3** Professor K demonstrates how to accomplish this.

Example Set 3

What percent of 800 is 200?

Step 1 Set up a proportion using information from the word problem.

$$\frac{200}{800}=\frac{n}{100}$$

In this problem the unknown number, *n*, is placed in the numerator of the rhs and the denominator is *100*.

Step 2 You must cross multiply to find the answer. Therefore, the following equation results:

$$\frac{200}{800}=\frac{n}{100}\rightarrow n\times 800=200\times 100$$

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Step 3 The n must be isolated on the lhs. *800* must divide both sides of the equation to accomplish the isolation of the n.

$$\frac{n\times 800}{800}=\frac{200\times 100}{800}\rightarrow n=\frac{20,000}{800}=25$$

Step 4 Simplify the rhs (divide *20,000* by *800*) to find that $n = 25$. Then, placing the *25* in percent form, you find that 200 is 25% of 800.

Notice in the original question (What percent of 800 is 200?) that the number following the word "of" (*800*) was placed in the denominator of the first fraction. The number following the word "is" (*200*) was placed in the numerator of the same fraction. *Percents* always have a *100* in the denominator of the fraction, therefore, the second fraction has a denominator of *100* and the numerator is the letter, n, because n represents the unknown quantity.

$$\frac{\text{is}}{\text{of}}=\frac{\%}{100}$$

As a general rule, always place the quantity after the word, "of" in the denominator of one ratio and the quantity after the word, "is" in the numerator of the same fraction. The part of a percent is placed in the numerator of another ratio and 100 is placed in the denominator of the same ratio. The two ratios are then separated with an equal sign to make a proportion. The quantity that is unknown is represented by a letter such as n and is placed in the proper location according to the general rule. Therefore, the easiest way to remember how to work any kind of percent problem is to memorize the following proportion or formula and substitute given numbers.

$$\frac{\text{is}}{\text{of}}=\frac{\%}{100} \qquad \text{In this case: } \frac{\text{is (200)}}{\text{of (800)}}=\frac{n}{100}$$

Work the activities for this lesson to demonstrate for Professor K your understanding of problems involving percents.

Life Principle



"People who plan to succeed often do. People who don't plan usually don't succeed."

—Dr. Ronald Johnson

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Basic Math Skills

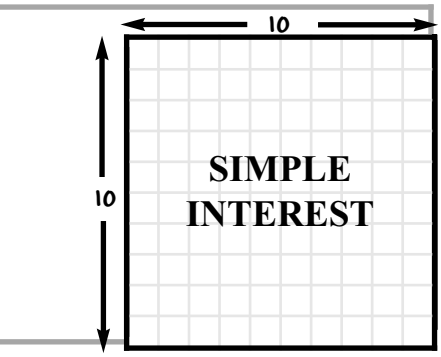
Chapter 2

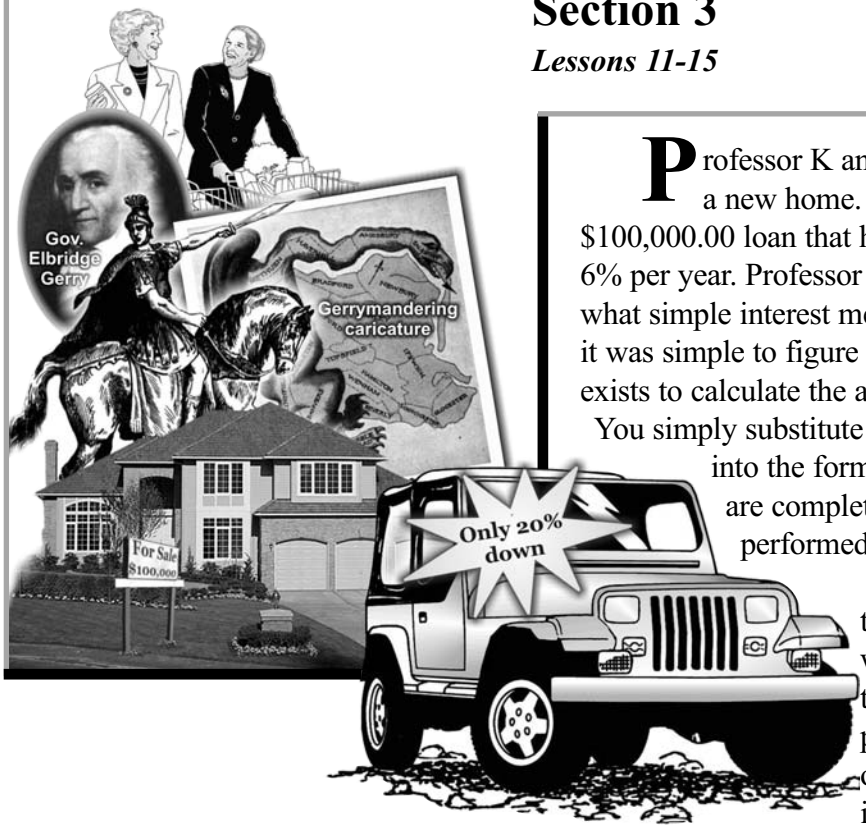
Lesson 15

Percent

Section 3

Lessons 11-15





Professor K and his wife wanted to purchase a new home. Professor K acquired a $100,000.00 loan that had a ***simple interest*** rate of 6% per year. Professor K's wife wanted to know what simple interest meant. Professor K told her that it was simple to figure simple interest. A formula exists to calculate the amount of interest to be paid. You simply substitute or "plug" the proper numbers into the formula. Once the substitutions are completed, the calculations are performed easily.

Mrs. K did not think that figuring simple interest was as simple as Professor K thought it was. Professor K proceeded to ***enlighten*** her on the calculation of simple interest.

Definition: Simple Interest Formula

Figuring simple interest is a method to determine the amount of interest owed on a loan or earned on an ***investment***. Simple interest is calculated using the following formula. $I = P \times r \times t$ where I = interest, P = ***principal***, r = rate (or percentage), and t = time

VOCABULARY

Simple Interest: a percentage of money charged by a money lender on the amount of money borrowed
Enlighten: to make a thing known to someone
Investment: to put money into a project expecting an income or profit
Principal: amount of money borrowed, excluding interest

Professor K's first explanation is contained in **Example Set 1.** Follow the explanation so that you can understand how to calculate simple interest.

Example Set 1

Professor K planned to borrow $100,000.00 to purchase his new home. The loan company was going to charge 6% interest per year. Calculate the amount of interest Professor K will pay the loan company the first year.

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Step 1

Use the interest formula to calculate the interest the first year by substituting the information provided into the formula.



$I = P \times r \times t$

Principal, P is the amount of the loan, $100,000.00

rate, r is rate of interest, 6% or .06 (move decimal two places to the left)

time, t is the amount of time to be calculated in years, one year (first year)

Step 2

Substitute the values from Step 1 into the simple interest formula and perform the calculations.

$I = 100{,}000 \times .06 \times 1 = 100{,}000 \times .06 = 6{,}000$

The amount of interest that Professor K will *pay* the loan company the first year is $6,000.00.

Look at **Example Set 2** as Professor K demonstrates calculating another simple interest calculation.

Example Set 2

Professor K had invested his savings so he could *earn* interest. Calculate the amount of simple interest earned on an investment of $50,000.00 after 2 years at an interest rate of 4%.

Step 1

Look at the information given to determine the values for each portion of the simple interest formula, $I = P \times r \times t$.

$P = \$50{,}000.00$; $r = 4\%$ or .04 (move the decimal two places to the left); $t = 2$

Step 2

Substitute the values into the simple interest formula.

$I = \$50{,}000.00 \times .04 \times 2 = \$50{,}000.00 \times .08 = \$4{,}000.00$

Therefore the simple interest earned on an investment of $50,000.00 after 2 years at a 4% interest rate is $4,000.00.

Professor K wants you to know that learning how to do interest calculations can assist you in your own life as you desire to invest or borrow money. Demonstrate your knowledge of simple interest by working the activities for this lesson.

Life Principle



"Wise people invest their time wisely."

—Anonymous

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Basic Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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